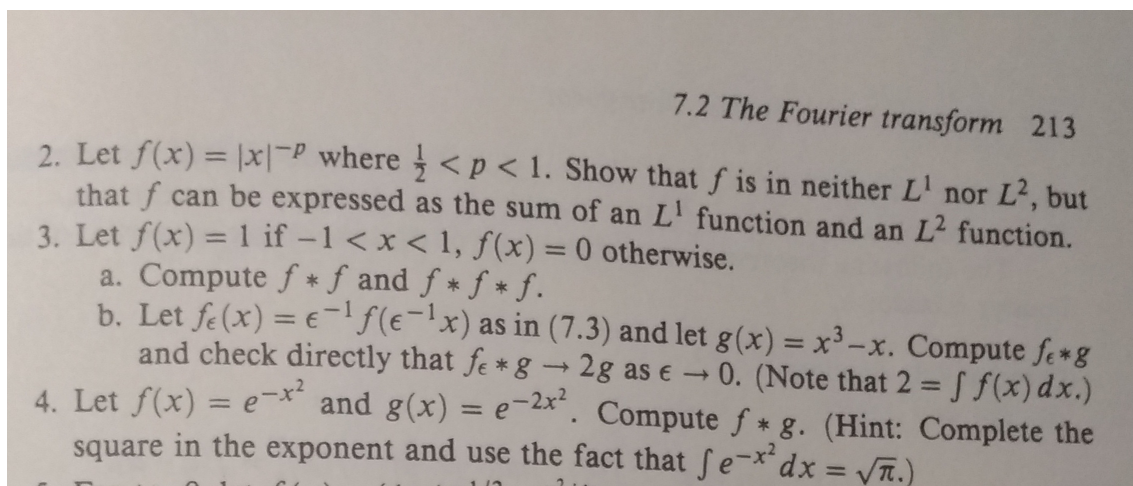


M 393C/CSE 396 Homework 1

Due Thursday, September 15, 2016

1. Folland Section 7.1, problems 2,3, and 4 (copied below).



2. Suppose $f, g \in L^1$. Prove the following Fourier transform formulas:
 - (a) If $f \in C^1(\mathbb{R})$ and $f'(x) \rightarrow 0$ as $|x| \rightarrow \infty$, then $\mathcal{F}\{f'\}(\xi) = (i\xi)\hat{f}(\xi)$.
 - (b) $\mathcal{F}\{f * g\}(\xi) = \hat{f}(\xi) \cdot \hat{g}(\xi)$
3. Show that for any $f \in L^2$ and any $\delta > 0$, there is a function g such that
 - (i) g is of class C^∞ , (ii) g vanishes outside a finite interval, and (iii) $\|f - g\| < \delta$. Proceed by the following steps:
 - Let $F(x) = f(x)$ if $|x| < N$, $F(x) = 0$ otherwise. Show that $\|F - f\| < \frac{1}{2}\delta$ if N is sufficiently large
 - Show that $g = F * K_\epsilon$ does the job if ϵ is sufficiently small and K is given by $K(y) = C^{-1}e^{-1/(1-y^2)}$ for $|y| < 1$, $K(y) = 0$ for $|y| \geq 1$.
4. Prove that the *sinc* function, $\text{sinc}(x) := \frac{\sin(\pi x)}{\pi x}$, $x \in \mathbb{R}$, belongs to $L^2(\mathbb{R})$, but not to $L^1(\mathbb{R})$.
5. Folland Section 7.2, problems 5 and 7 (copied below).

5. Suppose $g \in L^1$, $\int g(x) dx = 1$, and $\widehat{g} \in L^1$.
- Show that $\widehat{g}(\delta\xi) \rightarrow 1$ as $\delta \rightarrow 0$ for all $\xi \in \mathbf{R}$.
 - Show that for any continuous $f \in L^1$,

$$\lim_{\delta \rightarrow 0} \frac{1}{2\pi} \int e^{i\xi x} \widehat{g}(\delta\xi) \widehat{f}(\xi) d\xi = f(x)$$

for all x . What if f is only piecewise continuous? (Mimic the argument leading to (7.15), using the Fourier inversion theorem for g .)

- Show that $\int_0^\infty x^{-1} |\sin x| dx = \infty$. (Hint: Show that $\int_{(n-1)\pi}^{n\pi} x^{-1} |\sin x| dx > 2/n$.)
- Suppose that f is continuous and piecewise smooth, $f \in L^2$, and $f' \in L^2$. Show that $\widehat{f} \in L^1$. (Hint: First show that $\int (1 + \xi^2) |\widehat{f}(\xi)|^2 d\xi$ is finite; then use the Cauchy-Schwarz inequality as in the proof of Theorem 2.3, §2.3.)
- Given $a > 0$, let $f(x) = \frac{1}{x} \sin ax$. Show that $f \in L^2$ but $f' \notin L^2$.