M408D First Midterm Exam, October 3, 2002

1. A sequence $\{a_n\}$ is said to "grow faster" than $\{b_n\}$ if $\lim_{n\to\infty} b_n/a_n = 0$. Put the following sequences in order of growth rate, from fastest to slowest. JUSTIFY YOUR ANSWERS.

$a_n = \sqrt{3^n + 5^n}$,
$b_n = \ln(n^{1000})$,
$c_n = \sqrt{n}$,
$d_n = (n!)^{1/100}$,
$e_n = \ln(5e^n)$.

2. Evaluate the following limits or improper integrals, or write DNE if the limits do not exist.

a) $\displaystyle\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{1/x}$

b) $\displaystyle\int_0^\infty xe^{-x}dx$

c) $\displaystyle\lim_{x\to 0}\frac{1-\cos(x)}{\sin^2(x)}$

d) $\displaystyle\lim_{x\to 5}\frac{\sqrt{x^2-25}}{x^2-6x+4}$

3. Which of the following series converge, and which diverge. In each case, give a 1-sentence explanation (e.g., "converges by ratio test", or "diverges by comparison to 2^k")

a) $\displaystyle\sum \frac{k}{k^2+5}$

b) $\displaystyle\sum \frac{k^{15}}{2^k}$

c) $\sum k!e^{-k}$

d) $\displaystyle\sum \frac{\cos(\pi k)}{k}$

4. a) Write down the Taylor series for e^x.

b) Write down the 6-th order Taylor polynomial (i.e., through the x^6 term) for $f(x) = e^{-4x^3}$.

c) Evaluate $f^{(6)}(0)$. (That is, the 6-th derivative of $f(x)$, evaluated at $x = 0$.)

d) Evaluate $\int_0^{0.1} f(x)dx$ to five decimal places. (No, you don't need a calculator!)

5. a) Find the second-order Taylor polynomial for $f(x) = \sqrt{x}$ around $x = 4$.

b) What is the radius of convergence of the power series $\sum_{k=0}^{\infty}\frac{2k^2+17}{2^k}x^k$? Justify your answer.