

M408D Second Midterm Exam, November 5, 2002

1. Planes:

- a) Find the equation of the plane that goes through the point $(2,1,0)$ and has normal vector $(2,1,-2)$.
- b) Find the distance from the point $(4,13,5)$ to the plane in part (a).
- c) Find the equation of the plane that goes through the three points $(2,1,0)$, $(3,1,4)$ and $(0,0,0)$ [No, this isn't the same plane as part a)].
- d) Find the cosine of the angle between the planes of part (a) and (c).

2. Lines

- a) Find the equation, in symmetric form, for the line through the point $(2,1,0)$ in the direction $(2,1,-2)$.
- b) How far is the origin $(0,0,0)$ from this line?
- c) Find the equation, in symmetric form, for the line through the two points $(2,1,0)$ and $(3,-1,4)$.
- d) Find the equation (in symmetric form) of the line through $(2,1,0)$ that is perpendicular to the lines of parts (a) and (c).

3. Parametrized curves: Consider the parametrized curve

$$\mathbf{r}(t) = \left(\frac{t^2}{2}, \frac{4}{3}t^{3/2}, 2t + 5 \right).$$

- a) Compute the position, velocity, unit tangent vector and speed at time $t = 1$.
- b) Compute the arc-length of the curve from $t = 0$ to $t = 2$.

4. Polar coordinates.

- a) Sketch the curve $r = 1 + \cos(2\theta)$. Mark clearly the angles (if any) where the curve goes through the origin, and the angles where r is maximal.
- b) Find (in polar coordinates!) the points where this curve intersects the circle $r = 1/2$.
- c) Write down a definite integral that gives the area, in the first quadrant, inside the curve $1 + \cos(2\theta)$ but outside the circle $r = 1/2$.

Extra credit) Evaluate this integral.

5. Partial derivatives. Consider the function of two variables $F(x, y) = x^3y + 2x + y^2$.

- a) Compute $\partial F/\partial x$ and $\partial F/\partial y$ and evaluate these at the point $(1,1)$.
- b) Use these to estimate the value of $F(1.01, 1)$ and the value of $F(1, 1.01)$.
- c) Estimate the value of $F(1.02, 1.01)$.
- d) Compute the second-order partial derivatives $\partial^2 F/\partial x^2$, $\partial^2 F/\partial y^2$, and $\partial^2 F/\partial x\partial y$.