M346 First Midterm Exam, September 21, 2000

The exam is closed book, but you may have a single hand-written 8.5×11 crib sheet. There are 4 problems, each worth 25 points. I hope to have the exam returned to you next Thursday.

Good luck!

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1. Let V be the subspace of $\mathbb{R}_3$ consisting of polynomials $\mathbf{p}$ with $\mathbf{p}(0) = 0$. Let $\mathbf{b}_1 = -t + t^2$, $\mathbf{b}_2 = t + t^2 + t^3$, $\mathbf{b}_3 = -7t - 5t^2 + 2t^3$. Is the set $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ linearly independent? Does it span V? Is it a basis for V?

2. In $\mathbb{R}_2[t]$, let $\mathbf{b}_1(t) = 1 + t + t^2$, $\mathbf{b}_2(t) = 2 + 3t + t^2$, $\mathbf{b}_3(t) = 1 + 2t + t^2$, and $\mathbf{v}(t) = 5 - 2t + 3t^2$. Let $\mathcal{E} = \{1, t, t^2\}$ be the standard basis. Find $P_{\mathcal{E}\mathcal{B}}$, $P_{\mathcal{B}\mathcal{E}}$, and $[\mathbf{v}]_{\mathcal{B}}$.

3. On $\mathbb{R}_3[t]$, let L be the linear operator that shifts a function over to the left by one. That is, $(L\mathbf{p})(t) = \mathbf{p}(t+1)$. Find the matrix of L relative to the standard basis $\{1, t, t^2, t^3\}$.

4. In $\mathbb{R}_2[t]$, let $\mathbf{b}_1(t) = 1 + t + t^2$, $\mathbf{b}_2(t) = 2 + 3t + t^2$, $\mathbf{b}_3(t) = 1 + 2t + t^2$, as in problem 2. Let $L = d/dt$ be the derivative operator. Find the matrix of L relative to the basis $\mathcal{B}$. [You may find your answers to problem 2 to be useful.]