EXAM 2 REVIEW

Note: This review sheet is NOT meant to be a comprehensive overview of what you need to knowfor the exam. It is merely another tool to help you get started studying. The following concepts may or may not be seen on the exam and there may be concepts on the exam which are not coveredon this sheet.

General Advice:

· Only write something on your formula sheet once you understand it completely! Having many formulas that you don't understand how to use will only serve to confuse you during the exam.

· Many of the concepts we covered have more than one set of notation associated with them. For example, a vector can be written using the $\mathbf{v} =< x, y, z >$ notation, or the $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ notation. Also, see the box on page 948 of your textbook for all the different notations used for partial derivatives. Use whichever notation you are most comfortable with, but make sure that you can recognize all the different notations.

· Look over past homework assignments to recall previous problems and see if there are any specific concepts you don't understand . Also look over your first exam and correct any mistakes.

Chapter 14: Vector Functions

· Limits; Derivatives; Integrals: Vector functions are functions of the form $\mathbf{r}(t) =< f(t), g(t), h(t) >$. You input a scalar t into the equation $\mathbf{r}(t)$, and the output is a vector $< f(t), g(t), h(t) >$. To take limits (or derivatives, or integrals) of a vector function, just apply the limit (or derivative, or integral) to each component function of the output vector $< f(t), g(t), h(t) >$.

· Arc length; Curvature; Normal and Binormal Vectors: There is a nice summary of the equations introduced in this section boxed on page 904 of your textbook.

· Motion in Space: Understand vector functions describing position, velocity, and acceleration. How do you get velocity and acceleration from the position vector function? How do you get position and velocity from the acceleration vector function? What is the difference between velocity and speed?

Review Problems: from chapter 14:

(13) Find the curvature of the curve $y = x^4$ at $(1, 1)$.
(17) A particle moves with position function $\mathbf{r}(t) = t \ln t\mathbf{i} + t\mathbf{j} + e^{-t}\mathbf{k}$. Find vector equations for the velocity, speed, and acceleration of the partible.
(20) Find the tangential and normal components of the acceleration vector of a particle with position function $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$.
(8) Find the length of the curve traced out by the position equation $\mathbf{r}(t) =< 2t^{3/2}, \cos 2t, \sin 2t >$ as t ranges from 0 to 1. *Note: you must use u substitution to compute the integral.*
(1) Sketch the curve in the xy-plane given by the position function $\mathbf{r}(t) = t\mathbf{i} + \cos \pi t\mathbf{j} + \sin \pi t\mathbf{k}$ for $t \geq 0$. Find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.
(2) Let $\mathbf{r}(t) =< \sqrt{2-t}, (e^t - 1)/t, \ln(t + 1) >$. Find the domain of $\mathbf{r}(t)$, find the limit of $\mathbf{r}(t)$ as t approaches 0, and find $\mathbf{r}'(t)$.

Chapter 15: Partial Derivatives

· Functions of Several Variables: Recall the difference between a *function* and the *graph* of a function.

· Level Curves: How do you find the level curve of a surface? What does this mean geometrically? List a few ways that we've used level curves in class so far.

· Limits and Continuity: How do we check whether the limit of a function of **one** variable exists at a point? What does it mean for the limit of a function of two variables to exist at a point? What does continuity at a point imply about the limit of the function at that point?

· Partial Derivatives: Be very comfortable with these- you need them throughout ch.15

· Tangent planes; Linear approximation: In chapter 13 we learned several ways to write down the equations of planes and lines. Compare the equations given in chapter 15 for tangent planes to the equations we used to express a plane in chapter 13. Note similarities. Can you see why the chapter 15 equations work?

· Total Differential; Chain Rule: Let $z = f(x, y)$. Here z is a function of two variables, x and y. The total differential of z is $dz = (\partial d/\partial x)dx + (\partial d/\partial y)dy$. Put another way, the total change in d is the sum of the change in d caused by changing x and the change in d caused by changing y. Compare the formula for the total differential with the chain rule formulas. Can you see how the two are related? The section of chapter 15 entitled 'Chain Rule' gives several versions of the chain rule for functions of several variables. Compare the different versions.

· Directional Derivative: To calculate the directional derivative at a point, you need (1) a direction, **u** and (2) the gradient of the function at that point. In what direction does the maximum rate of change of a function occur? The directional derivative of f in the direction of **u** is written $D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$. So, is the directional derivative of f a scalar or a vector?

· The Gradient: Use the gradient (1) to describe the direction of the maximum rate of change of the directional derivative of a function and (2) to give a vector which is normal (orthogonal, perpendicular) to the tangent plane of a surface. Recall from chapter 13 that we used a point and a normal vector to define a plane. So given a point on the surface $z = f(x, y)$, we can easily write down the equation of a tangent plane using a point on the surface and the gradient evaluated at that point. Is the gradient evaluated at a point a scalar or a vector?

· Maxima and Minima: Be sure that you find all critical points when you solve $f_x = 0$ and $f_y = 0$. Remember, both these equations must be satisfied at a critical point. Understand the second derivative test for functions of two variables.

· Lagrange Multipliers: Write down the set of equations which must be satisfied according to the Lagrange multiplier method. Be sure you find *all* the points which satisfy your set of equations.

Professor Sadun suggested these review problems from chapter 15, page 1011: 10, 11, 12, 20, 27, 42, 46, 48, 52, 55, 61, 65.