M408M First Midterm Exam, October 5, 2006

1. Parametrized curves. Consider the curve $x = t^2$, $y = t - t^3/3$, with t running from 0 to 2.

a) Find the velocity (vector!) at time $t = 1$, and the slope of the curve at $t = 1$.

b) Find the length of the curve. (Yes, you CAN do this integral! This is very similar to a homework problem.)

2. Polar coordinates. Consider the curve C given by the formula $r = 3 + 2\cos(\theta)$.

a) Find the points of intersection, in polar coordinates, of this curve with the circle $r = 4$.

b) Find the Cartesian coordinates of the points you found in part (a).

c) Find the area of the region inside the curve C but outside the circle $r = 4$. [You get partial credit for setting up this integral, and full credit for solving it. The final answer does involve $\sqrt{3}$ and π.]

3. Equations of lines and planes.

a) Find the equation of the line through the point $(3, 2, 5)$ and parallel to the vector $(1, 2, -1)$. Express your answer FIRST in parametric form (or in vector form) and THEN in symmetric form.

b) Find the equation of the plane through the point $(3, 2, 5)$ perpendicular to the vector $(1, 2, -1)$.

4. Cosines.

a) Find the cosine of the angle between the line $x = y = z$ and the line $z = -y = z/2$.

b) Find the cosine of the angle between the plane $x + y + z = 17$ and the plane $x - y + 2z = 24$.

5. Triangles and planes. Consider the three points $P = (1, 2, 3)$, $Q = (2, 3, 4)$, $R = (3, 3, 3)$.

a) Find the area of the triangle PQR.

b) Find the equations of the plane through the three points P, Q and R.