

M408M Second Midterm Exam, November 9, 2006

1. Parametrized curves. A particle follows the curve  $\mathbf{r}(t) = \langle 3+2\cos(2t), 4+2\sin(2t), 3t+7 \rangle$ , where  $t$  represents time. Find
  - a) The speed of the particle at  $t = \pi$ .
  - b) The unit tangent vector  $T$  at time  $t = \pi$ .
  - c) The acceleration at time  $t = \pi$ , and
  - d) The distance traveled (arclength, not straight-line distance) from  $t = \pi$  to  $t = 2\pi$ .

Please put a BOX around each of your answers.

2. Partial derivatives and differentials. Consider the function  $f(x, y, z) = e^x(1 + \ln(y))z^2 + xy^2$ .
  - a) Compute  $f_x$ ,  $f_y$ , and  $f_z$  as functions of  $(x, y, z)$ , and evaluate them at the point  $(0, 1, 3)$ .
  - b) Use the results of part (a) to estimate the value of  $f(0.01, 0.98, 3.02)$ .
3. Suppose that  $f(x, y, z)$  is a function of three variables, and that  $f(1, 4, 3) = 13$ ,  $f_x(1, 4, 3) = 2$ ,  $f_y(1, 4, 3) = 5$ , and  $f_z(1, 4, 3) = -2$ .
  - a) Find the directional derivative of  $f$  at the point  $(1, 4, 3)$  in the direction of the point  $(3, 6, 4)$ . [Note that  $(3, 6, 4)$  is a point, not a vector.]
  - b) Find the equation of the tangent plane to the surface  $f(x, y, z) = 13$  at the point  $(1, 4, 3)$ .
4. Max-min.
  - a) Find all critical points of the function  $f(x, y) = x^2y^2 - 4x^2 - y^2 + 17$ .
  - b) Which of these critical points are local maxima? Minima? Saddle points?
5. Lagrange multipliers. Find the maximum (or maxima, if there's a tie) of the function  $f(x, y) = x^3 + y^3$  on the circle  $x^2 + y^2 = 2$ . Also, find the minimum (or minima). [Note: you do not have to classify the critical points as local maxima or minima. You just have to find the overall maximum and minimum values, and the points where these values occur.]