Problem 1. Political committees

13 Democrats, 12 Republicans and 8 Independents are sitting in a room. 8 of these people will be selected to serve on a special committee.

a) How many different possibilities are there for the committee membership?

b) What is the probability that exactly 5 of the committee members will be Democrats?

c) What is the probability that the committee will consist of 4 Democrats, 3 Republicans and one independent?

Problem 2. Number theory

a) How many solutions exist to the equation $x + y + z = 15$, where $x$, $y$, and $z$ have to be non-negative integers? Simplify your answer as much as possible. [Note: the solution $x = 12, y = 2, z = 1$ is not the same as $x = 1, y = 2, z = 12$]

b) How many solutions exist to the equation $xyz = 3^6 2^{13}$, where $x$, $y$ and $z$ have to be positive integers?

Problem 3. Spring Break Drinking

On South Padre Island over spring break, 25 underage students try to buy beer with fake IDs. Each has a 10% chance of getting caught, independent of the others.

a) What is the probability that exactly 4 of the students get caught?

b) What is the probability that 3 or more students get caught?

c) Evaluate your answer to part (b) numerically.

Problem 4. Grab bag

a) How many 9-letter license plates can be made by rearranging the letters of the phrase “I HATE MATH”?

b) 25 children submit science fair projects. The judges will award a 1st-place trophy, a 2nd place trophy, a 3rd place trophy, a 4th place trophy, a 5th place trophy, five identical “honorable mention” ribbons (for the 6th–10th best projects), and participation ribbons for the remaining children. In how many different ways can the judges distribute the prizes?