M362K Third Midterm Exam. April 11, 2002

1. Means and Variances

Let X be a continuous random variable with pdf

$$f(x) = 4xe^{-2x}, \qquad x > 0$$

(and zero if x is negative). For this problem, you might find the identity $\int_0^\infty t^n e^{-t} dt = n!$ useful.

a) Find $E(X)$.

b) Find the variance of X.

c) Find the probability that $X < 1$.

2. Joint continuous random variable Let X be a uniform random variable on the interval $[0, 1]$, and let Y be a uniform random variable on the interval $[0, 2]$. (In other words, $f_X(x) = 1$ for $0 < x < 1$, and $f_Y(y) = 1/2$ for $0 < y < 2$.) Suppose that X and Y are independent random variables.

a) Write down the joint pdf $f_{X,Y}(x, y)$.

b) Let $Z = X + Y$. Find the pdf $f_Z(z)$. Simplify as much as possible.

c) Find the expectation $E(X)$ and variance $Var(X)$. Repeat for Y.

d) Compute the expectation $E(Z)$ and the variance $Var(Z)$.

3. A silly game. In the (fictional) game of "dice-flip", each player flips a coin and rolls one die. If the coin comes up tails, his score is the number of dots showing on the die. If the coin comes up heads, his score is twice the number of dots on the die. (E.g., (tails,4) is worth 4 points, while (heads,3) is worth 6 points.) Let X be the first player's score.

a) Compute the pdf $f_X(x)$ for all numbers x.

b) Compute the cdf $F_X(x)$ for all numbers x.

c) Find the probability that $X < 4$. Is this the same as $F_X(4)$?

4. Calendar follies. A month of the year is chosen at random (each with probability 1/12). Let X be the number of letters in the month's name, and let Y be the number of days in the month (ignoring leap year). [For the record, January has 31 days, February has 28, March has 31, April has 30, May has 31, June has 30, July has 31, August has 31, September has 30, October has 31, November has 30 and December has 31.]

a) Write down the joint pdf of X and Y. From this, compute the pdf of X and the pdf of Y. (You may want to compile a single table with all this information).

b) Find $E(Y)$.

c) Are the events "$X \leq 6$" and "$Y = 30$" independent?

d) Are X and Y independent random variables?