M362K Final Exam, December 9, 2010

1. The lottery.

A state lottery involves the state randomly picking 5 (different) numbers between 1 and 40. Specifically, there are 40 balls, and five of them are drawn from a bin.

a) What is the probability that all five numbers are even?

b) What is the probability that 27 is the 2nd number chosen?

c) What is the probability that 27 is the 2nd largest number chosen?

2. Sock drawer

Joey has a very messy sock drawer, containing two white socks, two black socks, two red socks, and 16 other socks, none of which match. In his rush to get dressed, he pulls four socks at random from the drawer in the hope of getting two that match.

a) What is the probability of his drawing a pair of white socks?

b) What is the probability of his drawing at least two matching socks?

3. Teenage drivers

On evenings when my daughter Rina drives, there is a 1/1000 chance of her getting into an accident. When my son Allan drives, there is a 3/1000 chance of an accident. When Rina and Allan go out together, Rina drives 60% of the time and Allan drives 40% of the time.

a) If Rina and Allan go out together tonight, what is the probability of there being an accident?

b) If there is an accident, what is the probability that Rina was driving?

4. Shooting baskets.

Every time I shoot a foul shot in basketball, I have a 1/3 chance of scoring.

a) If I shoot 5 shots, what is the probability of scoring 3 times?

b) If I shoot for a long time, what is the probability that the first score will be on the 3rd attempt?

c) If I shoot for a long time, what is the probability that the second score will be on the 5th attempt or later. Simplify your answer as much as possible.

5. Transforming random variables

Let X be a continuous random variable whose pdf if $f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$.

a) Compute the cdf $F_X(x)$, the expectation $E(X)$, and the variance $Var(X)$.

b) Let $Y = X^2$ and let $Z = \sqrt{X}$. Compute the pdf's of Y and of Z.

6. Dreidels

Today is the last day of Hannukah. A traditional holiday game involves spinning a top (called a dreidel) with four sides. There are four possible outcomes, each equally likely: you can win the pot (let's say 10 coins), you can win half the pot (5 coins), you can lose 3 coins, or nothing can happen. (The coins are typically chocolate, BTW)

a) Let X be your winnings from one spin of the top. Compute the expectation and variance of X.

b) Let Y be the net winnings from 200 spins of the top. Compute the mean and standard deviation of Y.

c) What is the probability that Y is at least 558?

7. Joint distributions

Let X and Y be continuous random variables with the joint pdf $f_{X,Y}(x, y) = 3|x - y|$ when x and y are both between 0 and 1, and zero otherwise.

a) Compute the marginal pdf's $f_X(x)$ and $f_Y(y)$. [Hint: To handle the absolute value, break your region of integration into two pieces depending on the sign of $x - y$.]

b) Compute the covariance $Cov(X, Y)$.

c) Let $Z = X + Y$. Compute the pdf $f_Z(z)$.

8. Grading exams

Let X be the amount of time (in minutes) that it takes to grade an exam question, and suppose that X is uniformly distributed between 1 and 3 minutes. Note that this is a continuous random variable. Also suppose that the time it takes to grade different papers (or different questions on the same paper) are independent of each other.

a) Compute the cdf $F_X(x)$ for all real values of x, the expectation $E(X)$ and the variance $Var(X)$.

b) Compute the (approximate) probability of grading 300 questions in 615 minutes or less.