M362K First Midterm Exam, September 29, 2010

1. The Texas Lottery is considering a new game in which players pick 7 (different) integers between 1 and 40 and the state then randomly picks 5 (different) integers between 1 and 40, say by pulling 5 balls out of a rotating bin. If your choices include all 5 of the state's numbers, you win the jackpot.

a) Suppose that the winning numbers one week are 3, 5, 18, 22, and 38. How many different player choices can win the jackpot?

b) In a different week, suppose that you pick 2, 3, 5, 8, 13, 21, and 34. How many different possibilities for the state's numbers will lead to your winning the jackpot.

c) If you play once, what is the probability of your winning the jackpot?

2. Two baseball teams (call them A and B) are tied for first place with 7 games left in the season. Their last 7 games are against each other, so whichever team wins the majority of games will win the division.

Suppose that team A has a 50-50 chance of winning each game, independent of all the other games.

a) What is the conditional probability that A wins the division, given that they win the first game? Simplify your answer as much as possible.

b) What is the conditional probability that A wins the first game, given that they win the division?

3. The safety record of Delaware Overseas Airways (DOA) is not the best. On any given flight, each engine has a 1/3 chance of failure, independent of any other engine. Fortunately, it only takes one good engine to fly a 2-engine plane, and it only takes two good engines to fly a 4-engine plane. (If *more* than half the engines fail, then the plane will crash and everybody on the plane will die.)

a) If you fly on a 2-engine plane, what is the probability that you will survive the flight?

b) If you fly on a 4-engine plane, what is the probability that you will survive the flight?

4. Suppose that I have two coins in my pocket, one regular and one 2-headed. I pull a coin out at random (50% chance for each), and then flip it twice. Let E be the event that the first toss is heads. Let F be the event that the second toss is heads. Are E and F independent? Why or why not?

5. A number, like 3 or 1221 or 353, that reads the same forwards and backwards, is called a pallindrome. How many pallindromes are there between 1 and 1,000?