M362K Second Midterm Exam, November 3, 2010

1. Baseball

Major league baseball is considered switching to a shorter regular season with an expanded playoff system. Suppose that in the new league, the season runs for 144 games, and that team A has a 50% chance of winning each game it plays, independent of all the other games.

a) (10 pts) Approximate the probability that team A wins 80 games or more.

b) (5 pts) Give an *exact* expression for the probability that team A wins exactly 72 games.

c) (10 pts) Give a good numerical approximation for the probability that team A wins exactly 72 games. (Your answers to (a) and (c) should be numbers like 0.135, not expressions like $\binom{8}{3}e^{-5.4}$.)

2. Being nickled and dimed.

Suppose that I independently flip three nickels and three dimes. The nickels are weighted so that they each have a 2/3 chance of coming up heads. The dimes are weighted so that they each have a 1/3 chance of coming up heads. Let X be the number of nickels that come up heads, let Y be the number of dimes that come up heads, and let $Z = X + Y$.

a) (9 pts) Compute the probability mass function p_X, the expectation $E(X)$ and the variance $Var(X)$.

b) (6 pts) Compute the joint probability mass function $p_{X,Y}$.

c) (10 pts) Compute the probability mass function p_Z.

3. A continuous distribution

Suppose that the pdf of a variable X is given by

$$f_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{2} & \text{if } -1 < x < 0 \\ \frac{e^{-x}}{2} & \text{if } x \geq 0 \end{cases}$$

Compute the cdf $F_X(x)$, the expectation $E(X)$ and the variance $Var(X)$. You may find the identity $\int_0^\infty x^n e^{-x} dx = n!$ to be useful.

4. Fishing for answers

In a lake by the French Mathematical Institute, fishing is a Poisson process, with each fisherman catching an average of 0.5 fish per hour.

(a) (15 pts) If Pierre spends 10 hours fishing one day, what is the probability of his coming home without any fish? What is the probability that he will come home with 3 or more fish?

(b) (10 pts) If 100 fishermen each spend 10 hours fishing, what is the (approximate) probability that at least two of them will come home without fish? (You may wish to express the answer to part (b) in terms of your answers to part (a)).