M362K First Exam. October 8, 1996

Note that this test was given about 2 weeks farther into the term than we are now. As a result, some of the questions are about Chapter 3 material, and you shouldn’t expect to be able to do them yet.

**Problem 1. Teddy Bears**

My daughter Rina has lost her favorite teddy bear. From past experience, I estimate that there is a 2/3 chance it is in her room somewhere, and a 1/3 chance that it is in her brother’s room. If the bear *is* in her room and she looks hard, she has a 3/5 chance of finding it. I tell her to go back to her room and look hard.

a) What is the probability of her finding the bear?

b) Let’s suppose Rina comes out of her room a few minutes later, still missing her bear. What is the probability that the bear is in her brother’s room, given that she failed to find it in her room?

**Problem 2. The Texas Lottery**

In the “Pick Six” game in the Texas Lottery, you pick 6 numbers between 1 and 40, the state picks 6 numbers, and if 3 of your numbers match you win “an average of $25”. (If you match more than 3 you win more money, but in this problem we’ll just talk about the odds of matching 3). [Disclaimer: All I know about the lottery is what I see in the TV ads. I may well have misrepresented the “Pick Six” game. If so, just pretend that the game is played by my rules.]

a) Each time you play Pick Six, what is the probability of your matching exactly 3 of the winning numbers? (You can leave your answer in terms of factorials or binomial coefficients).

b) Suppose you play the numbers 1-2-3-4-5-6 and a friend plays 7-8-9-10-11-12. What is the probability that at least one of you matches (exactly) 3 of the winning numbers?

**Problem 3. Continuous distributions**

Suppose I pick a number $x$ at random (i.e. by the uniform distribution) between 0 and 2. Suppose I pick $y$ at random between 0 and 4.

a) Describe the sample space. Draw a picture of the sample space. In your picture, show the event “$y < x^2$”.

b) Find the probability that $y < x^2$.

**Problem 4. Breakdowns**
A system consists of 4 components (call them A, B, C, and D). On a given day, each component has a probability $p$ of failing (independent of the others). If 3 or more components fail, the system breaks down.

a) What is the probability of the system breaking down today? (Leave your answer in terms of $p$, but otherwise simplify as much as possible).

b) Given that the system broke down yesterday, what is the probability that component A failed?