M362K First Exam. October 8, 1996

Note that this test was given about 2 weeks farther into the term than we are now. As a result, some of the questions are about Chapter 3 material, and you shouldn't expect to be able to do them yet.

Problem 1. Teddy Bears

My daughter Rina has lost her favorite teddy bear. From past experience, I estimate that there is a 2/3 chance it is in her room somewhere, and a 1/3 chance that it is in her brother's room. If the bear *is* in her room and she looks hard, she has a 3/5 chance of finding it. I tell her to go back to her room and look hard.

a) What is the probability of her finding the bear?

b) Let's suppose Rina comes out of her room a few minutes later, still missing her bear. What is the probability that the bear is in her brother's room, given that she failed to find it in her room?

Problem 2. The Texas Lottery

In the "Pick Six" game in the Texas Lottery, you pick 6 numbers between 1 and 40, the state picks 6 numbers, and if 3 of your numbers match you win "an average of $ 25". (If you match more than 3 you win more money, but in this problem we'll just talk about the odds of matching 3). [Disclaimer: All I know about the lottery is what I see in the TV ads. I may well have misrepresented the "Pick Six" game. If so, just pretend that the game is played by my rules.]

a) Each time you play Pick Six, what is the probability of your matching exactly 3 of the winning numbers? (You can leave your answer in terms of factorials or binomial coefficients).

b) Suppose you play the numbers 1-2-3-4-5-6 and a friend plays 7-8-9-10-11-12. What is the probability that at least one of you matches (exactly) 3 of the winning numbers?

Problem 3. Continuous distributions

Suppose I pick a number x at random (i.e. by the uniform distribution) between 0 and 2. Suppose I pick y at random between 0 and 4.

a) Describe the sample space. Draw a picture of the sample space. In your picture, show the event "$y < x^2$".

b) Find the probability that $y < x^2$.

Problem 4. Breakdowns

A system consists of 4 components (call them A, B, C, and D). On a given day, each component has a probability p of failing (independent of the others). If 3 or more components fail, the system breaks down.

a) What is the probability of the system breaking down today? (Leave your answer in terms of p, but otherwise simplify as much as possible).

b) Given that the system broke down yesterday, what is the probability that component A failed?