

M362K First Exam Solutions. October 8, 1996

Note that this test was given about 2 weeks farther into the term than we are now. As a result, some of the questions are about Chapter 3 material, and you shouldn't expect to be able to do them yet.

Problem 1. Teddy Bears

My daughter Rina has lost her favorite teddy bear. From past experience, I estimate that there is a $2/3$ chance it is in her room somewhere, and a $1/3$ chance that it is in her brother's room. If the bear *is* in her room and she looks hard, she has a $3/5$ chance of finding it. I tell her to go back to her room and look hard.

a) What is the probability of her finding the bear?

Let A be the event that the bear is in her room, and let B be the event that she finds it. $P(A)=2/3$, $P(B|A)=3/5$, and $P(B|\text{not } A)=0$, so $P(B)=P(A)P(B|A)+P(\text{not } A)P(B|\text{not } A)=(2/3)(3/5)+0 = 2/5$.

b) Let's suppose Rina comes out of her room a few minutes later, still missing her bear. What is the probability that the bear is in her brother's room, given that she failed to find it in her room?

$$P(A^c|B^c) = P((A^c \cap B^c)/P(B^c) = (1/3)/(3/5) = 5/9.$$

Problem 2. The Texas Lottery

In the "Pick Six" game in the Texas Lottery, you pick 6 numbers between 1 and 40, the state picks 6 numbers, and if 3 of your numbers match you win "an average of \$ 25". (If you match more than 3 you win more money, but in this problem we'll just talk about the odds of matching 3). [Disclaimer: All I know about the lottery is what I see in the TV ads. I may well have misrepresented the "Pick Six" game. If so, just pretend that the game is played by my rules.]

a) Each time you play Pick Six, what is the probability of your matching exactly 3 of the winning numbers? (You can leave your answer in terms of factorials or binomial coefficients).

The total number of possible drawings is $\binom{40}{6}$. The number of ways to match 3 of your 6 (and 3 of the remaining 34) is $\binom{6}{3}\binom{34}{3}$. So the answer is $\frac{\binom{6}{3}\binom{34}{3}}{\binom{40}{6}}$.

b) Suppose you play the numbers 1-2-3-4-5-6 and a friend plays 7-8-9-10-11-12. What is the probability that at least one of you matches (exactly) 3 of

the winning numbers?

Let A be the event that you match 3 and let B be the event that your friend matches 3. $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

$$\frac{\left[2 \binom{6}{3} \binom{34}{6} - \binom{6}{3}^2 \right]}{\binom{40}{6}}$$

Problem 3. Continuous distributions

Suppose I pick a number x at random (i.e. by the uniform distribution) between 0 and 2. Suppose I pick y at random between 0 and 4.

a) Describe the sample space. Draw a picture of the sample space. In your picture, show the event " $y < x^2$ ".

The picture should be the rectangle with corners (0,0), (2,0), (2,4) and (0,4). The event $y < x^2$ is everything under the parabola $y = x^2$.

b) Find the probability that $y < x^2$.

The area of the event is $\int_0^2 x^2 dx = 8/3$. The area of the total region is 8, so the probability of the event is $(8/3)/8 = 1/3$.

Problem 4. Breakdowns

A system consists of 4 components (call them A, B, C, and D). On a given day, each component has a probability p of failing (independent of the others). If 3 or more components fail, the system breaks down.

a) What is the probability of the system breaking down today? (Leave your answer in terms of p , but otherwise simplify as much as possible).

$$P(\text{breakdown}) = P(3 \text{ failures}) + P(4 \text{ failures}) = 4p^3(1-p) + p^4.$$

b) Given that the system broke down yesterday, what is the probability that component A failed?

The only scenario that doesn't involve A failing is if B, C, and D fail and A doesn't. This has probability $p^3(1-p)$. The remaining scenarios have probability $3p^3(1-p) + p^4$. So the conditional probability is

$$P(\text{A failed} | \text{breakdown}) = \frac{3p^3(1-p) + p^4}{4p^3(1-p) + p^4}.$$