Algebraic Topology

Homework 3: Due September 15

1. Page 42, problem 3.2

2. Page 46, problem 4.9. You may want to do 4.8 as a warm-up. Also, you should assume that S is *not* a sphere.

3. Page 49, problem 5.1

4. Page 50, problem 5.2

5. Page 50, problem 5.3.

Problem 6. Let X be any topological space. (If you want, you can restrict your attention to Hausdorff spaces, but it really doesn't matter.) let $Y = [0,1] \times X/\sim$, where $(0,x) \sim (0,y)$ for all $x, y \in X$. Y is called the *cone* of X, and the equivalence class of $(0,x)$ is called the *cone point*. Prove that Y is path-connected (easy) and simply connected.

Problem 7. Let X be any topological space, and let $Y = [0,1] \times X/\sim$, where $(0,x) \sim (0,y)$ and $(1,x) \sim (1,y)$ for all $x, y \in X$. Y is called the *(free) suspension* of X, and is sometimes denoted SX. (The *reduced suspension* ΣX is a slightly different space that doesn't concern us here. Look up Suspension (topology) on Wikipedia if you want the definition.) For instance, S^{k+1} is the suspension of S^k. Prove that if X is path-connected, then Y is simply-connected.