Algebraic Topology
Homework 8: Due Wednesday, October 20
Revised October 18

Problem 1 Back in early September, we proved that the fundamental group of a circle was infinite cyclic using Lebesgue numbers and an argument about angles. I want you to take that "keep track of angles" idea to its logical conclusion. Use the properties of the cover $R \to S^1$, $x \to e^{ix}$ and Lemmas 3.1, 3.2 and 3.3 to prove that $\pi_1(S^1) = Z$. Do *not* use Lebesgue numbers or partitions of the interval $[0, 1]$ into pieces (except insofar as these concepts were required to prove Lemmas 3.1-3.3)

Problem 2. Let X be a path-connected and locally path-connected Hausdorff space, and consider the category C_1 whose objects are covering spaces of X, and whose morphisms are homomorphisms of covering spaces (as defined on page 130). Show that C_1 need not have a universal object. [Hint: it's sufficient to consider $X = S^1$.]

Problem 3. Now pick a base point $x \in X$, and consider the category C_2 whose objects are *based* covering spaces $(\tilde{X}, \tilde{x}, p)$ where $p(\tilde{x}) = x$, and whose morphisms are required to take base points to base points. Show that $(\tilde{X}, \tilde{x}, p)$ is a universal object if $\tilde{X}$ is simply-connected. (The converse is also true but is harder.)

Problem 4 Page 132, problem 6.4

The following two problems were originally assigned, but are **no longer due on October 20**. They will be included in homework #8, due on October 27.

Problem 5 Page 135, problem 7.2 – For example 2.4, you can restrict attention to the cover $\mathbb{R}^2 \to T^2$.

Problem 6 Page 144, problem 10.1