Algebraic Topology

Homework 9: Due Wednesday, November 3

Problem 1. Page 163, problem 2.1

Problem 2. Page 166, problem 3.3. The hard part is the last statement.

Cone operators. Let X be a star-like subset of Euclidean space, meaning that if $x \in X$, then $tx \in X$ for every $t \in [0, 1]$.

As usual, let $C_n(X)$ be generated by the singular n-chains on X, modulo the degenerate chains. Let $C_{-1} = \mathbf{Z}$, with the usual augmentation map $C_0 \to C_{-1}$ that counts points. That is, we're computing *reduced* homology. Consider the *cone operator* $\phi : C_n \to C_{n+1}$ defined as follows. If $n \geq 0$ and T is an n-cube, let $\phi(T)(x_1, \ldots, x_{n+1}) = x_1 T(x_2, \ldots, x_{n+1})$. If $n = -1$, let $\phi(1)$ be the 0-cube that maps to the origin.

Problem 3. Show that $\partial \circ \phi + \phi \circ \partial$ is the identity on C_n. Use this fact to prove that the reduced homology of X is trivial.

Problem 4. Let Y be any topological space, and let X be the cone of Y. Modify the construction of Problem 1 to show that the reduced homology of X is trivial.

Problem 5. More generally, let X be any contractible space. Use an appropriate cone operator to show that the reduced homology of X is trivial.

Please do problems 3–5 in order. Don't just do problem 5 and then say "3 and 4 are special cases". They are, but the idea is to go from the concrete and specific to the abstract and general in stages.

Problem 6. As a continuation of last week's assignment on simplicial homology, see if you can generalize, finding (and proving!) a formula for the homology of Δ_0^k (including its faces and sub-faces). The key is constructing an appropriate cone operator, albeit in the category of simplicial complexes and simplicial chains rather that topological spaces and singular chains.