Algebraic Topology Midterm Exam, October 15, 2010

1. (30 points) (a) Let X be a punctured torus and let γ be a loop around the puncture, as depicted on the blackboard. Let x_0 be a point on γ. Compute $\pi_1(X, x_0)$ and express the class of γ in terms of the generators and relations of that group.

(b) Now let Y be the union of X with a disk, as shown on the board. Compute $\pi_1(Y, x_0)$ and compute the class of γ in $\pi_1(Y, x_0)$.

(c) Now let Z be the union of $T^2 \# T^2$ and two disks, as shown on the board. Compute $\pi_1(Z)$.

2. (20 points) The infinite dihedral group D_∞ is a group of symmetries of the real line generated by a translation $t(x) = x + 1$ and a reflection $r(x) = -x$. Show that D_∞ is isomorphic to the free product $\mathbb{Z}_2 * \mathbb{Z}_2$.