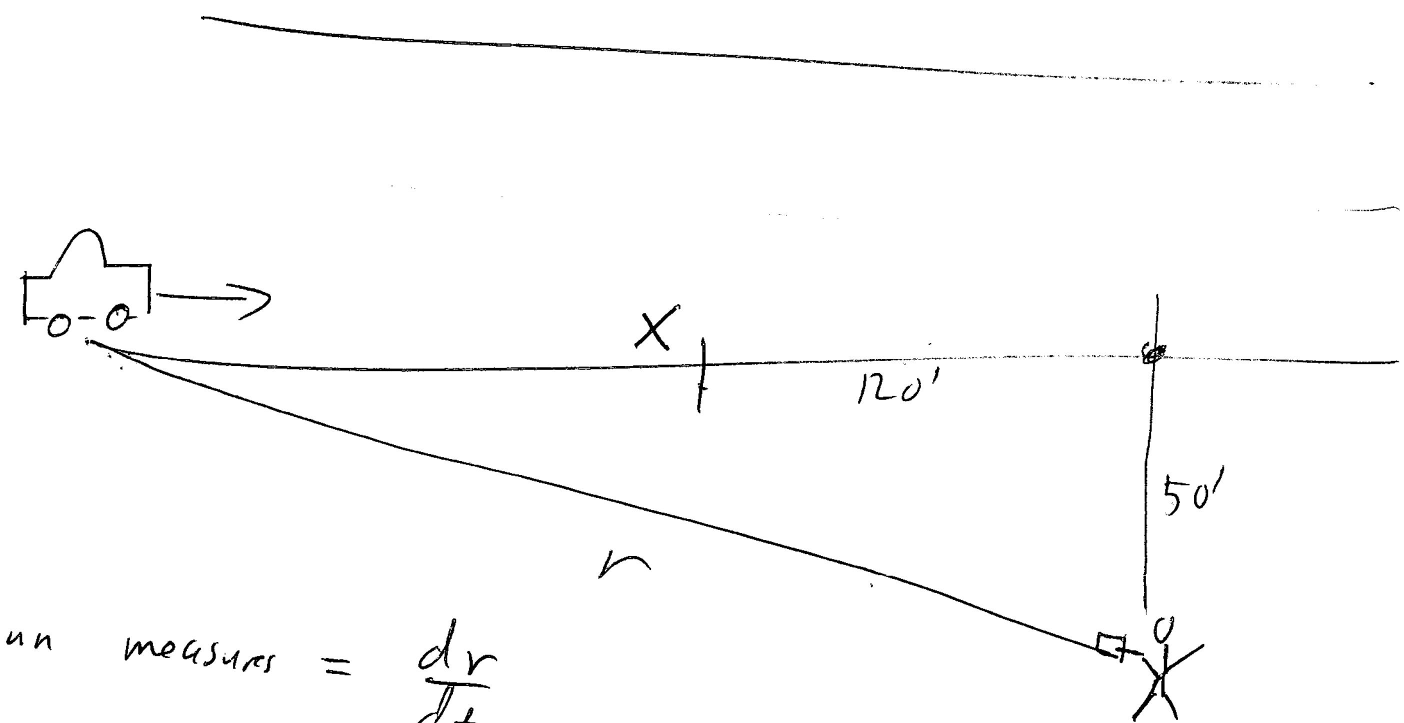
$\frac{dr}{dt} = -60$ mph when $x = 120$



Radar gun measures $= \frac{dr}{dt}$

$r^2 = x^2 + (50)^2$

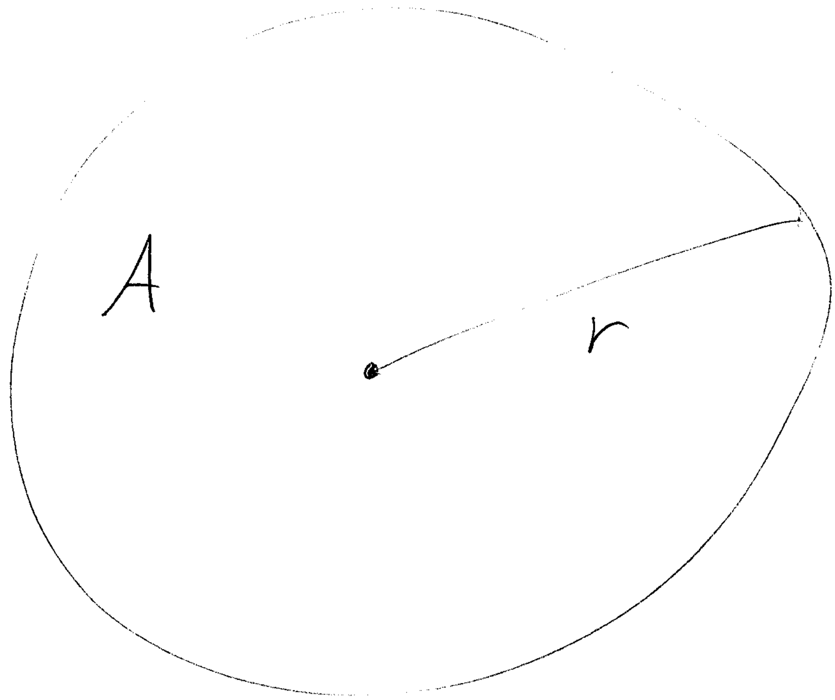
$2r\frac{dr}{dt} = 2x\frac{dx}{dt} + 0 \implies \frac{dx}{dt} = \frac{r}{x}\overset{-60}{\frac{dr}{dt}} = \frac{130}{120}(-60) = -65$

Given: Description of a problem.
Rate of change of one variable. z

Want: Rate of change of other variable. x

1) Write down what you know.
(draw picture!)
2) Derive relation between variables.
3) Take derivative w.r.t. <u>time</u>
(Get equation relating $\frac{dz}{dt}$ to $\frac{dx}{dt}$)
4) Solve for $\frac{dx}{dt}$.

Forest fire burns on circular region.
Radius increases by 1 mile/day $\left(\frac{dr}{dt}=1\right)$

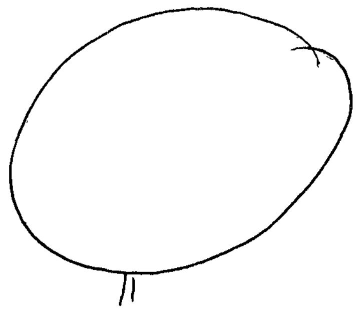


At what rate is the burned area increasing when radius = 5 miles?

$$A = \pi r^2 \qquad \cancel{\frac{dr}{dt} = 1}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi \underbrace{(5)}_{5\text{ miles}} \cdot (1)\ \frac{\text{mile}}{\text{day}}$$

$$= 10\pi \ \frac{(\text{miles})^2}{\text{day}}$$



Pumping up a balloon.

Pump at rate of

$\boxed{.5\ (ft)^3/min}$

At what rate is $\boxed{\text{radius increasing}}$

a) When $r = 1$ ft?

b) when $r = 10$ ft?

Variables are radius r

Volume V.

$$\frac{dV}{dt} = .5\ \frac{(ft)^3}{min}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

$$= \frac{1}{4\pi r^2}\cdot\frac{1}{2} = \frac{1\ (ft)^3/min}{8\pi r^2}$$

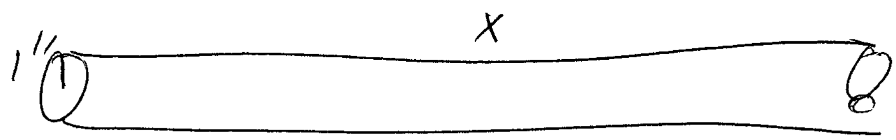
When $r = 1$ foot

$$\frac{dr}{dt} = \frac{1\ (ft)^3/min}{8\pi\cdot 1^2 (ft)^2} = \frac{1}{8\pi}\ ft/min$$

When $r = 10$ feet

$$\frac{dr}{dt} = \frac{1}{800\pi}\ ft/min$$

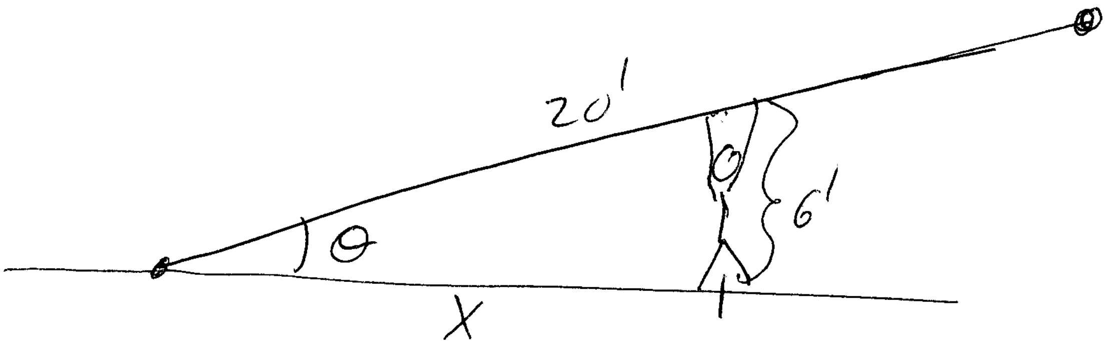
Make balloon animals



$V = \text{area} \cdot \text{length}$

$= \pi 1^2 \cdot x = \pi x$

$\frac{dV}{dt} = \pi \frac{dx}{dt}$; $\frac{dx}{dt} = \frac{1}{\pi} \frac{dV}{dt}$



Suppose $\frac{dx}{dt} = -2'/\text{sec}$

What is $\frac{d\theta}{dt}$ when $x = 8'$

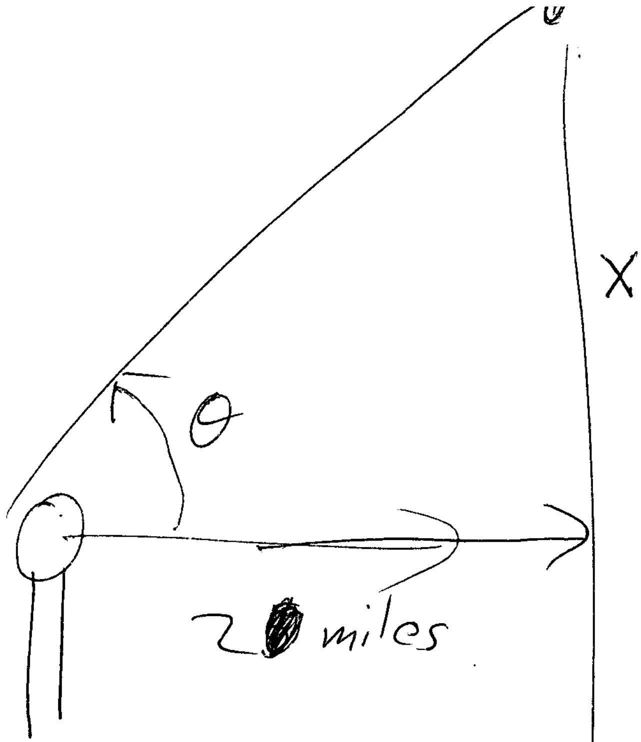
$$\tan\theta = \frac{6}{x}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{-6}{x^2}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = -\frac{6}{x^2}\cdot\frac{dx}{dt}\cdot\cos^2\theta = \frac{12}{x^2}\cos^2\theta$$

$$= \frac{12}{8^2}\cdot\left(\frac{4}{5}\right)^2 = .12$$

In general, $\frac{d\theta}{dt} = \frac{12}{x^2}\left(\frac{x}{\sqrt{x^2+6^2}}\right)^2 = \frac{12}{x^2+6^2}$



What is $\dfrac{dx}{dt}$ if $\dfrac{d\theta}{dt} = 1 \; \dfrac{\text{radian}}{\text{sec}}$

$$\tan\theta = \frac{x}{2}$$

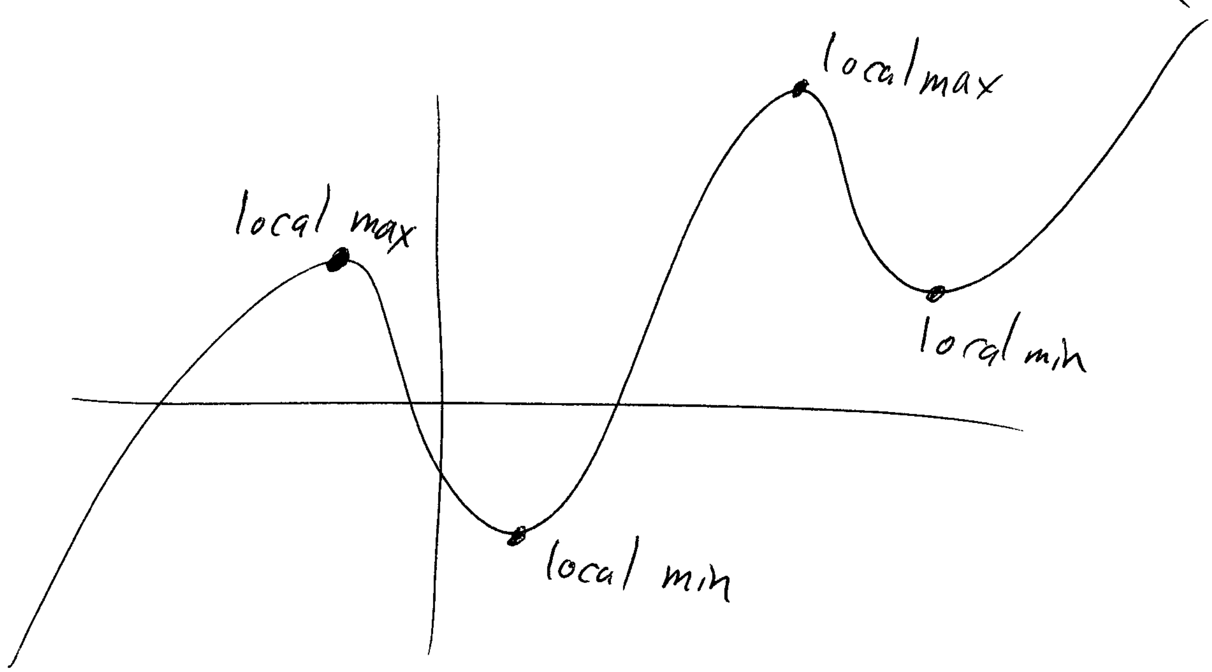
$$\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}\,\frac{1}{2}$$

$$\frac{dx}{dt} = 2\sec^2\theta \frac{d\theta}{dt} =$$

$$= 2\sec^2\theta \cdot 1$$

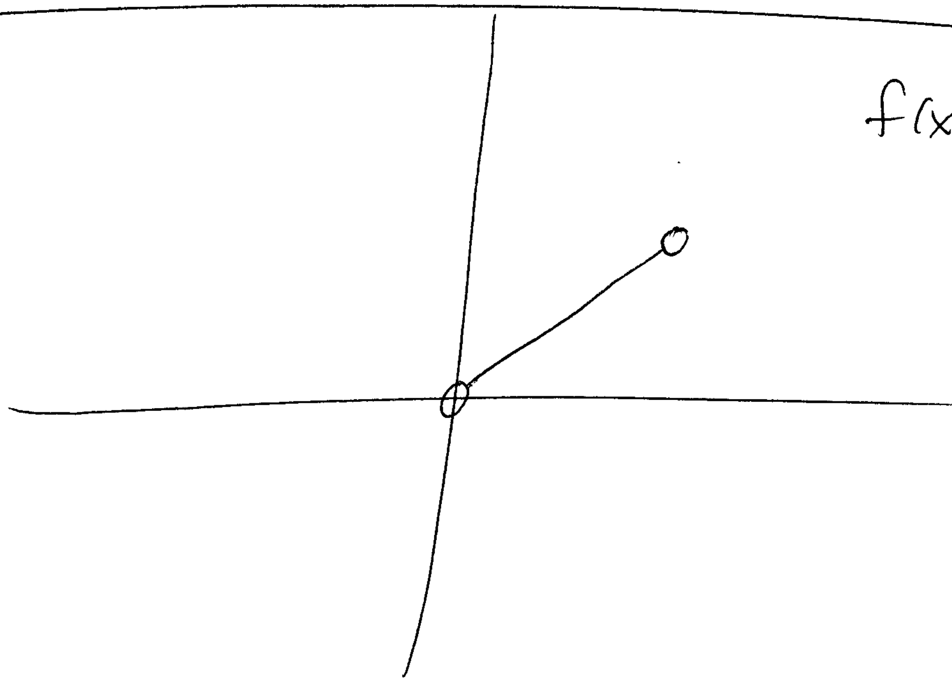
A function $f(x)$ has a <u>local maximum</u> at $x=a$ if $f(a) \geq f(x)$ for all x close to a.

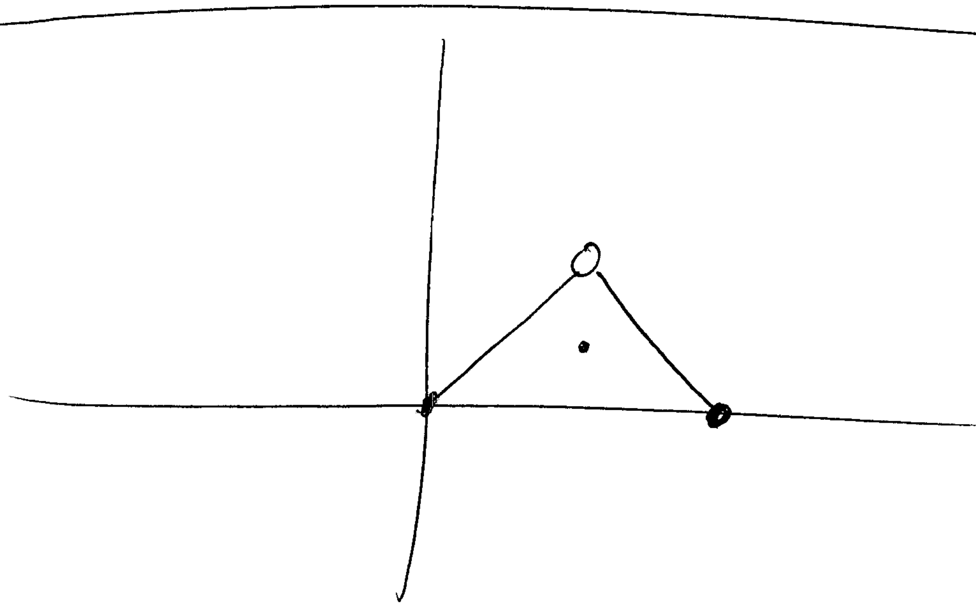
A function $f(x)$ has a <u>global maximum</u> at $x=a$ if $f(a) \geq f(x)$ for all x, (period)



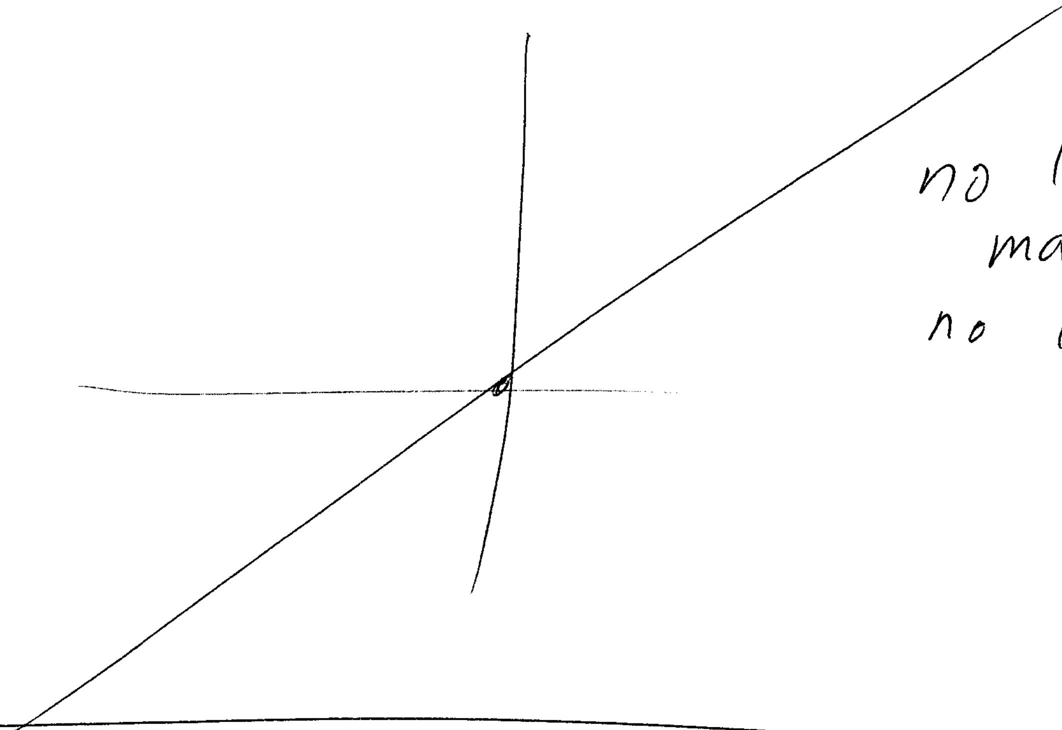
<u>Thm</u> If $f(x)$ is a continuous function on $[a,b]$, then ~~the~~ f has a global max on $[a,b]$.

$f(x) = x$ on $(0,1)$



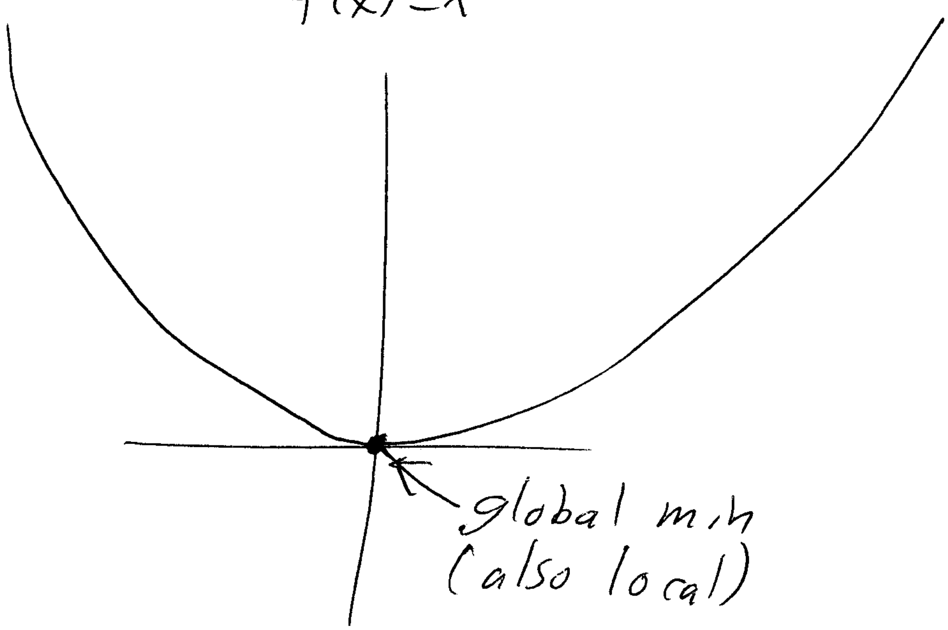


$f(x) = x$ on $\mathbb{R}$



no local/global max,
no local/global min..

$f(x) = x^2$



global min
(also local)