Midterm Thursday

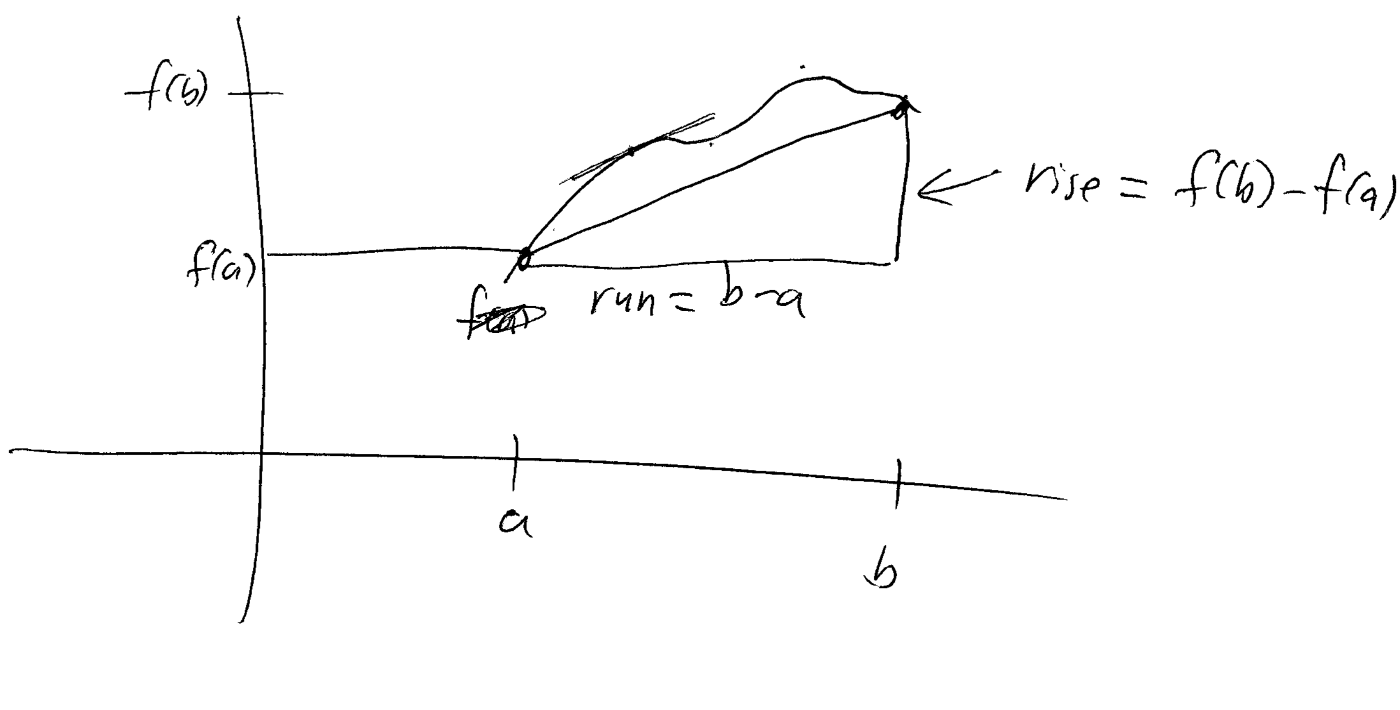
Covers through 4.2 (Skip 3.8, 3.11)

Format $\approx$ like last time

One crib sheet ok.

Rolle's Thm If f is continuous on $[a,b]$, differentiable on (a,b) and $f(a)=f(b)$, then there is a point c in (a,b) with $f'(c)=0$.

Mean Value Thm If f is cont. on $[a,b]$ and differentiable on (a,b), then there is a point c in (a,b) with $f'(c) = \dfrac{f(b)-f(a)}{b-a}$



$$g(x) = f(x) - \left(\frac{f(b)-f(a)}{b-a}\right)(x-a)$$

$$g(a) = f(a)$$

$$g(b) = f(b) - \left(\frac{f(b)-f(a)}{b-a}\right)(b-a)$$

$$= f(b) - (f(b) - f(a)) = f(a)$$

$$g'(c) = 0$$

$$g'(x) = f'(x) - \left(\frac{f(b)-f(a)}{b-a}\right) \cdot 1$$

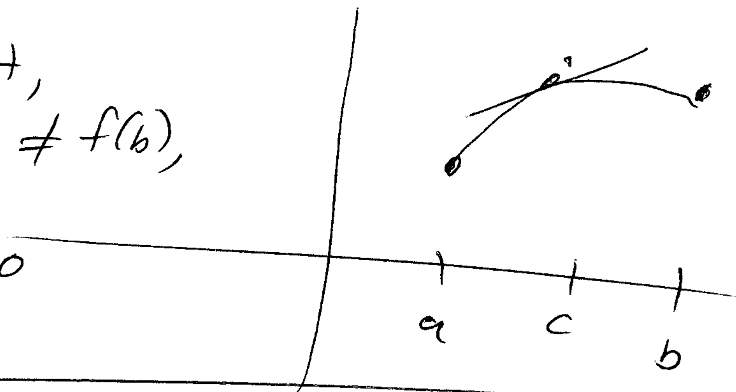
$$g'(c) = f'(c) - \left(\frac{f(b)-f(a)}{b-a}\right) = 0$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Thm

If $f'(x) = 0$ for all x, then $f(x) = \text{constant}$.

Pf If not, and if $f(a) \neq f(b)$, then $f'(c) \neq 0$



Thm If $f'(x) = g'(x)$ for all x, then $f(x) = g(x) + c$

pf : $(f-g)' = f' - g' = 0$, so $f - g = c$

If $f'(x) > 0$ on (a,b), then f is increasing on $[a,b]$.

(In other words, if $x_2 > x_1$, then $f(x_2) > f(x_1)$)

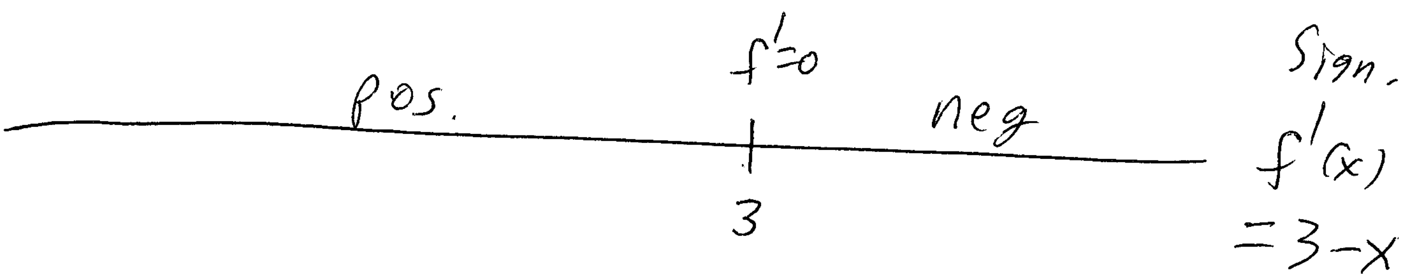
If $f' < 0$, f is decreasing.

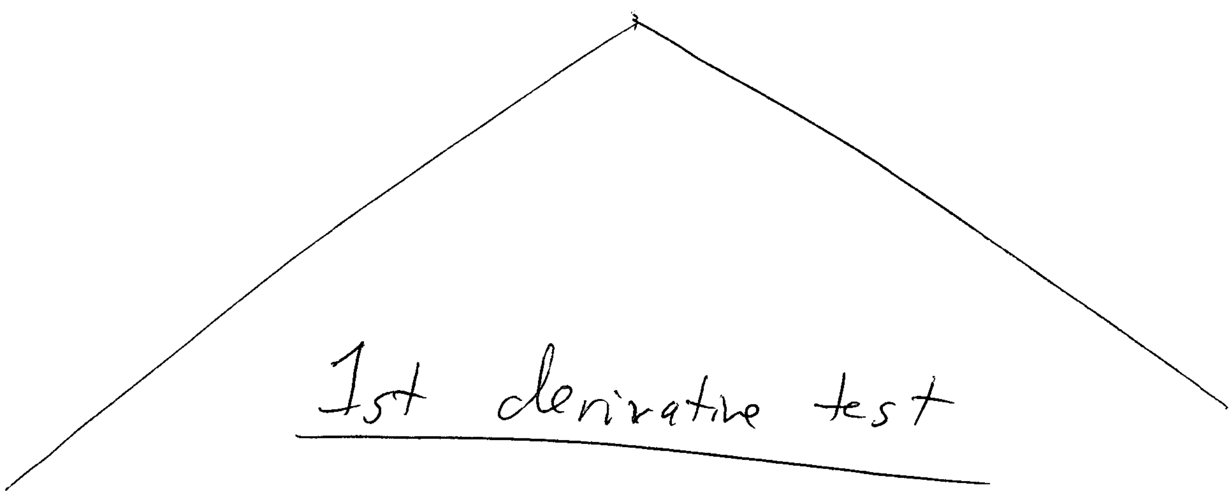
What does sign of f' tell you?

$f' > 0 \implies$ increasing

$f' < 0 \implies$ decreasing

$f'(c) = 0 \implies$ momentarily stopped.





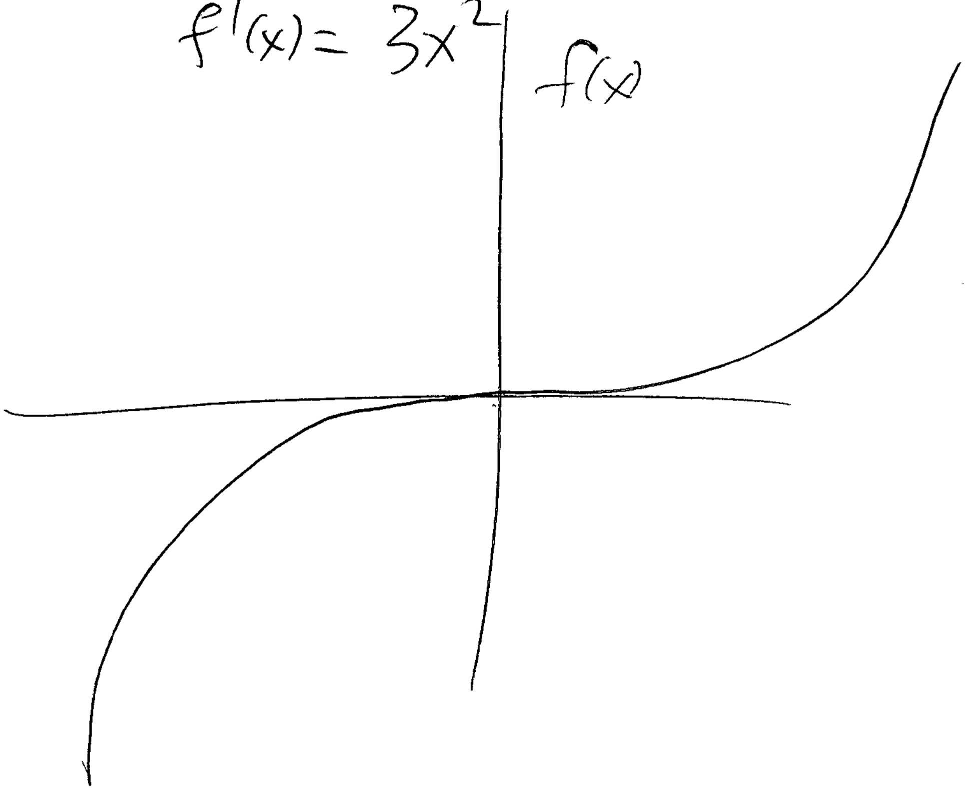
<u>1st derivative test</u>

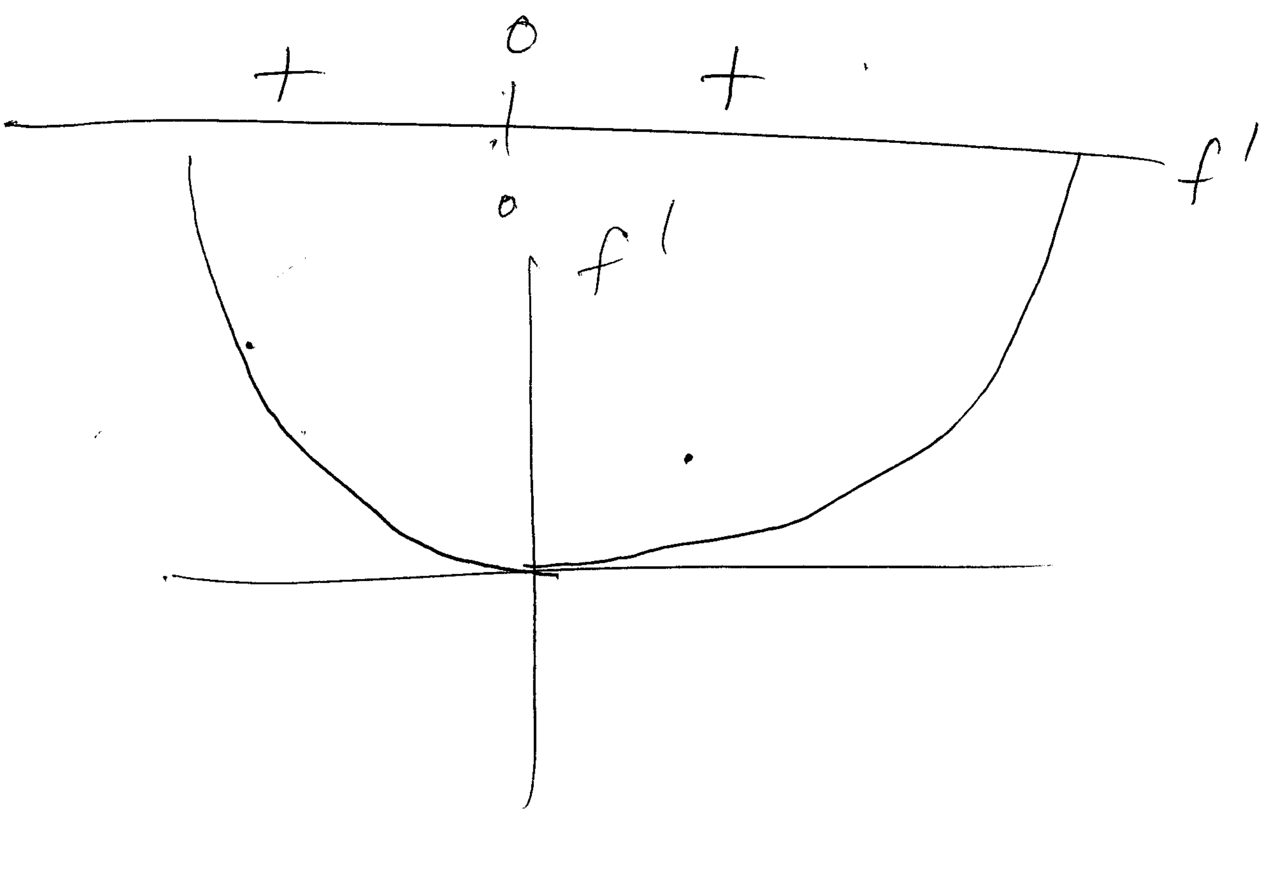
When f' goes from $+$ to $-$, f is at a local max

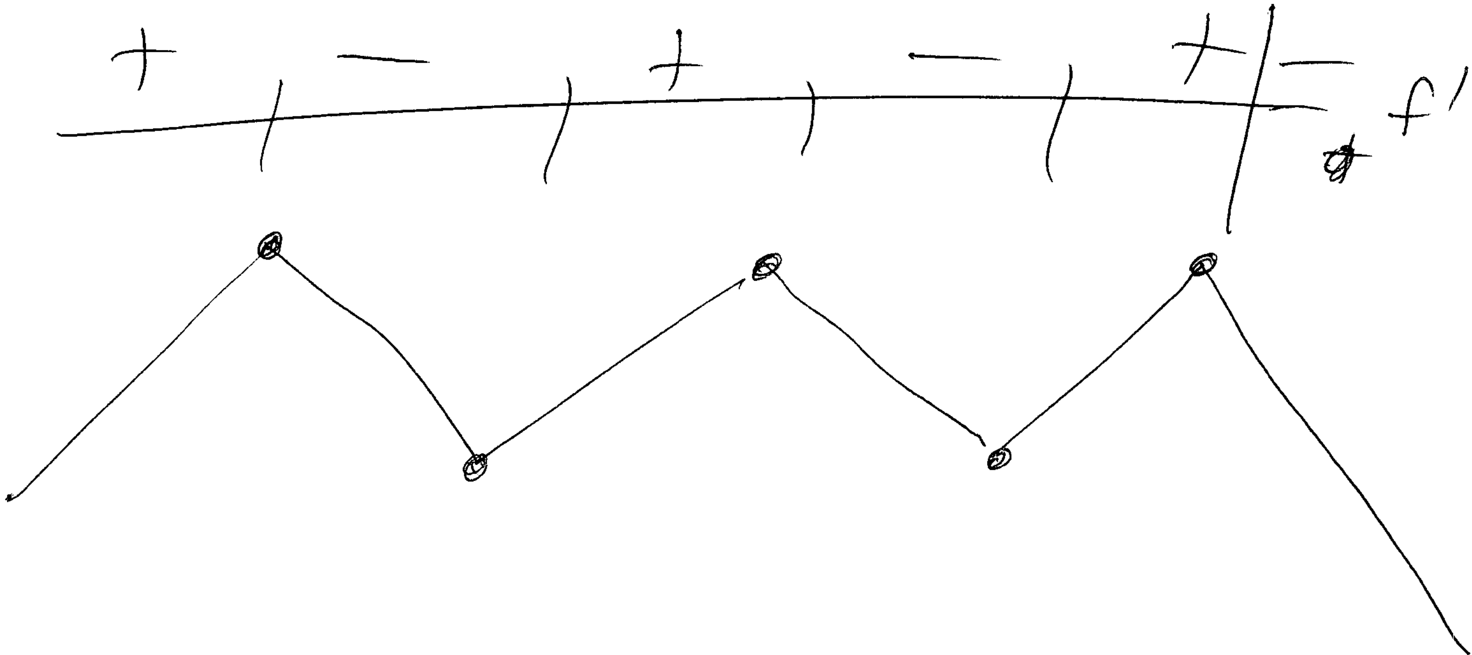
When f' goes from $-$ to $+$, local min.

$f(x) = x^3$

$f'(x) = 3x^2$






+ − + − + −
f'

$f'' > 0 \Rightarrow f'$ increasing.

$f' > 0, \ f'' > 0$

$f' < 0 \quad f'' > 0$

$f' = 0 \quad f'' > 0$

$f'' > 0 \Rightarrow$ curving upwards $\Leftrightarrow$ ~~Con~~ concave up.

2nd derivative test:

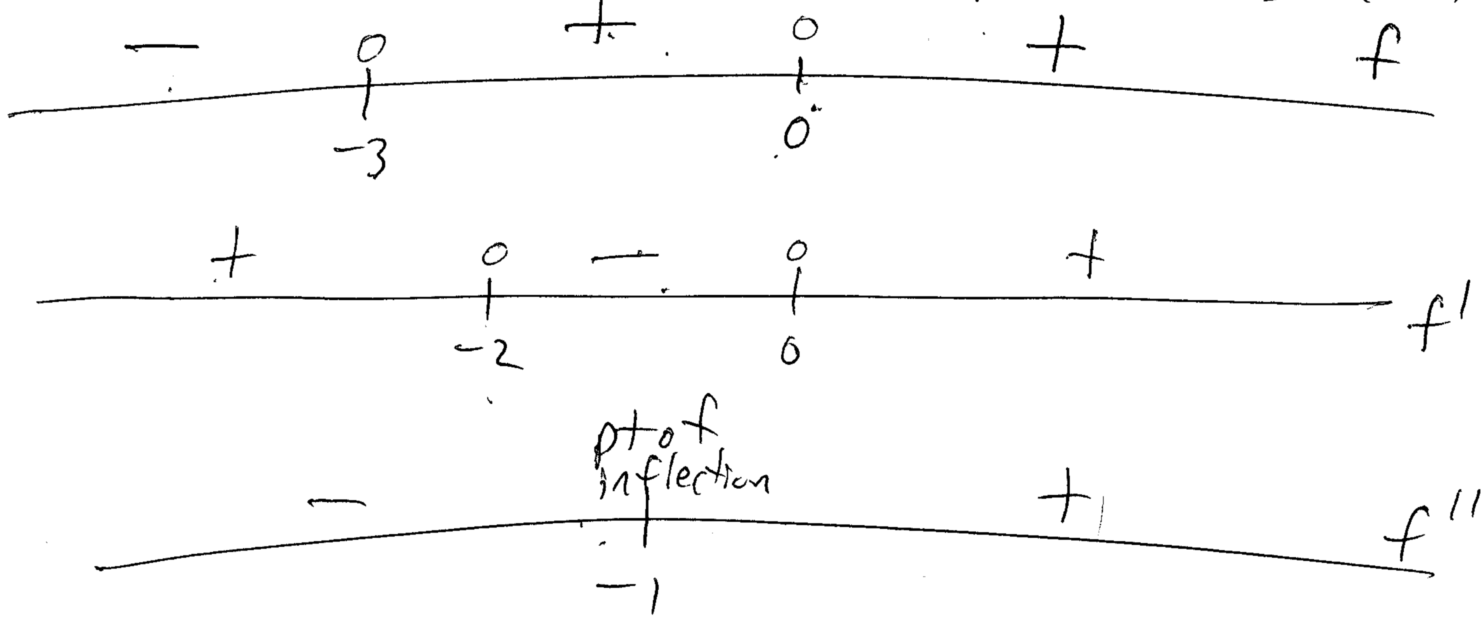
If ~~$f'(c)$~~ $f'(c) = 0$ and $f''(c) > 0$, c is a local min.

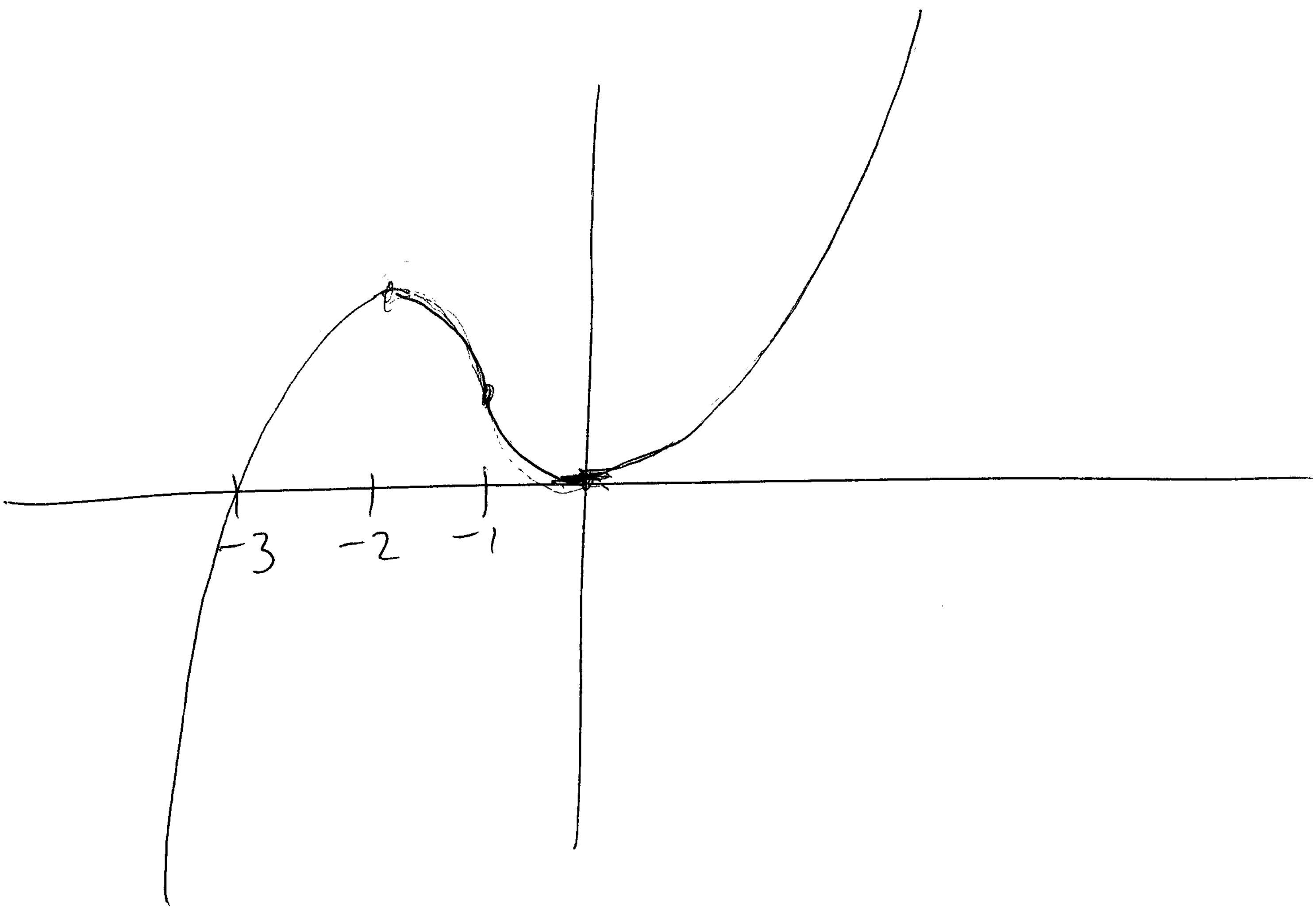
If $f'(c) = 0$ and $f''(c) < 0$, c is a local max.

$$f(x) = x^3 + 3x^2 = \underline{x^2}\,\underline{(x+3)}$$

$$f'(x) = 3x^2 + 6x$$
$$= 3x(x+2)$$

$$f''(x) = 6x + 6$$
$$= 6(x+1)$$





Derivatives of std functions

f	f'
x^n	nx^{n-1}
$\ln(x)$	$1/x$
e^x	e^x
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$

Products & quotients

$$(fg)' = (f')(g) + f(g')$$

ex.

$$\frac{d}{dt}\left(\sin(x)\, e^x\right) = \cos(x)\, e^x + \sin(x)\, e^x$$

$$= (\sin(x) + \cos(x))\, e^x$$

$$\left(\frac{f}{g}\right)' = \frac{g f' - f g'}{g^2}$$

Chain rule:

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

$$\frac{d}{dx} \sin(x^2) = \cos(x^2) \cdot 2x$$

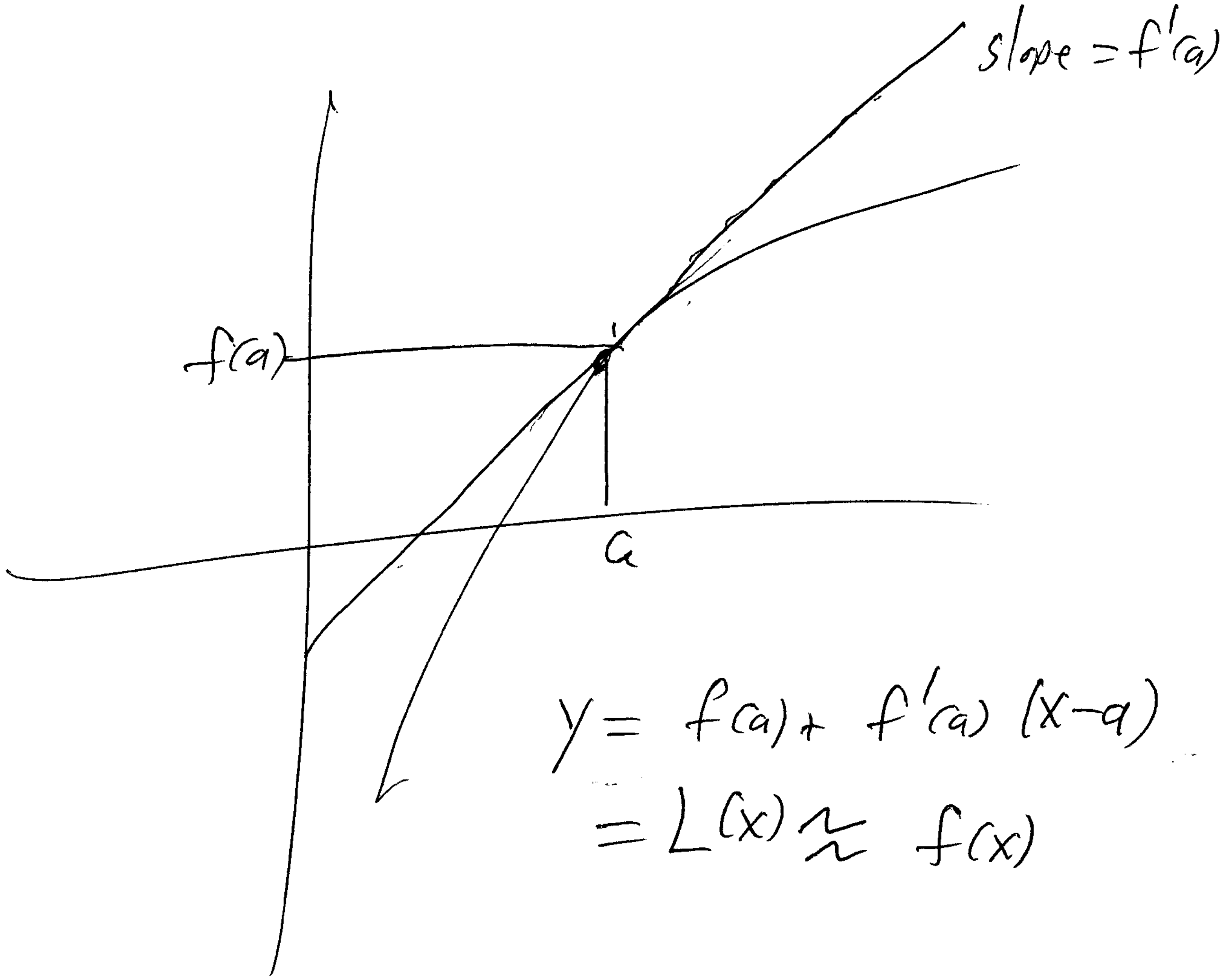
$$\left(\frac{x+1}{x^2-7}\right)^{15}$$

$$\left(\frac{\sin(e^x)}{\ln(15+x^4)}\right)^2$$

$$x^2y + \cancel{ye} \; xe^y = 17$$

$$2xy + x^2\frac{dy}{dx} + e^y + xe^y\frac{dy}{dx} = 0$$

$$2xy\frac{dx}{dt} + x^2\frac{dy}{dt} + e^y\frac{dx}{dt} + xe^y\frac{dy}{dt} = 0$$



$$y = f(a) + f'(a)(x-a)$$
$$= L(x) \approx f(x)$$