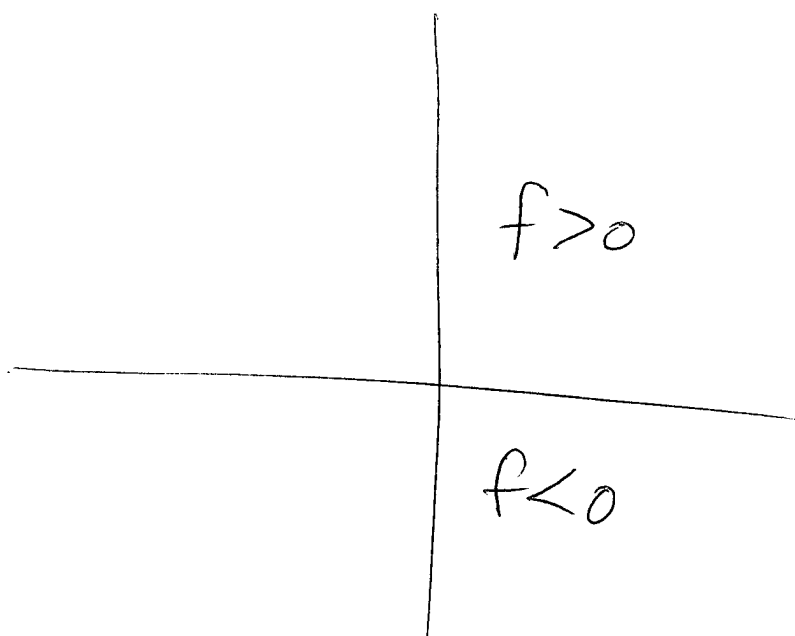


What does  $f$  tell you?

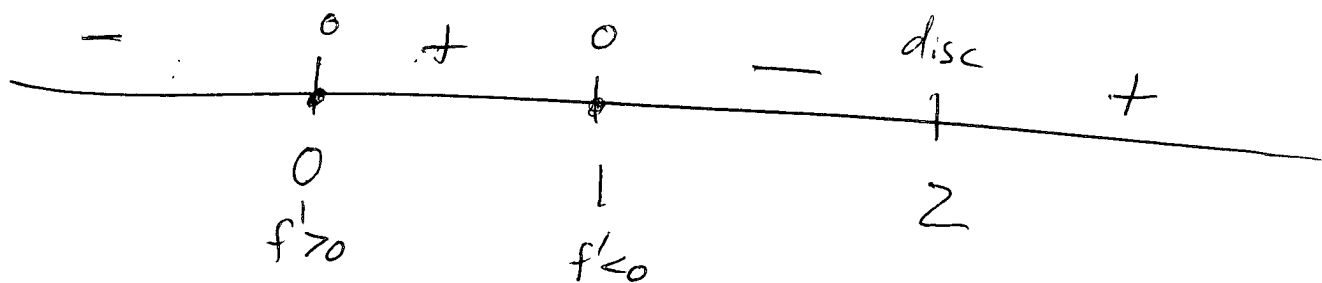
What does  $f'$  tell you?

What does  $f''$  tell you?



Look ~~for~~ for places where  $f=0$  or  $f$  discontinuous.

$$f(x) = \frac{x(x-1)}{x-2} = \frac{x^2-x}{x-2}$$



3 ways to tell sign

1) Factor.

2) Derivatives.

3) Test points

$$f'(x) = \frac{(x-2)(2x-1) - (x^2-x) \cdot 1}{(x-2)^2}$$

$$= \frac{2x^2 - 5x + 2 - x^2 + x}{(x-2)^2} = \frac{x^2 - 4x + 2}{(x-2)^2}$$

$$f'(0) = \frac{2}{4} = \frac{1}{2} > 0$$

$$f'(1) = \frac{1-4+2}{(1-2)^2} = \frac{-1}{1} = -1 < 0$$

If  $f(a) = 0$  and  $f'(a) > 0$ , then  
 $f(x) > 0$  to the right of  $a$  and  
 $f(x) < 0$  to the left.

If  $f(a) = 0$  and  $f'(a) < 0$ , then  
 $f(x) > 0$  to the left and  
 $f(x) < 0$  to the right.

If  $f(a) = 0$  and  $f'(a) = 0$ , can't tell.

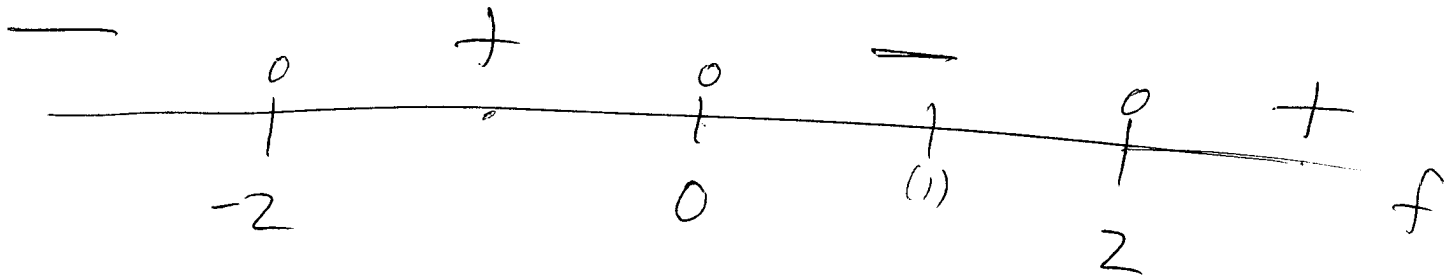
$$f(x) = x^3 - \frac{4}{3}x$$

$$f(1) = -3$$

$$f(3) = 15$$

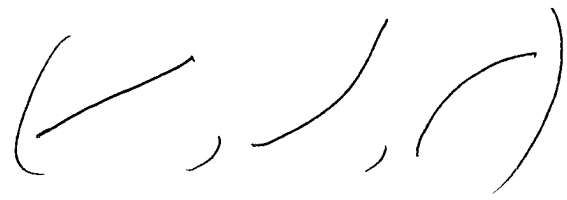
$$f(-1) = 3$$

$$f(-3) = -15$$



Look at sign of  $f'$

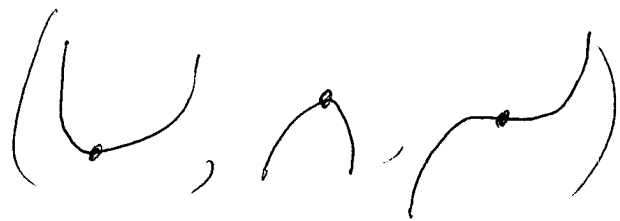
If  $f' > 0$ , increasing



If  $f' < 0$ , decreasing



If  $f' = 0$ , momentarily flat

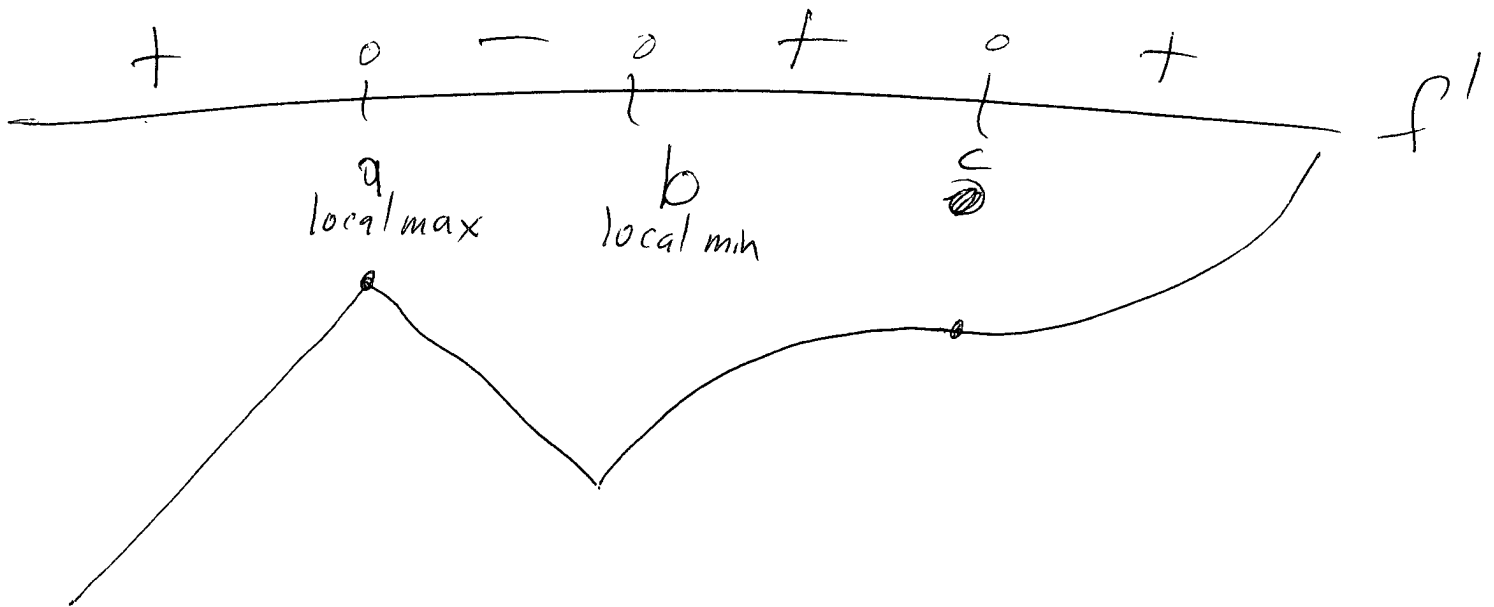


When  $f'$  goes from  $-$  to  $+$ ,  
local ~~max~~ min

---

When  $f'$  goes from  $+$  to  $-$ ,  
local ~~min~~ max

---



## 2nd derivative test

If  $f'(a) = 0$  and  $f''(a) > 0$ ,

(then  $f'(x)$  must go from  $-$  to  $+$  at  $a$ .)

So  $a = \text{local min}$

---

If  $f'(a) = 0$  and  $f''(a) < 0$ ,

local max

---

If  $f'(a) = 0$  and  $f''(a) = 0$ , can't tell.

---

$f'' < 0$  curving down  $f' > 0$   $f' < 0$



$f'' > 0$  curving up  $f' < 0$   $f' > 0$

