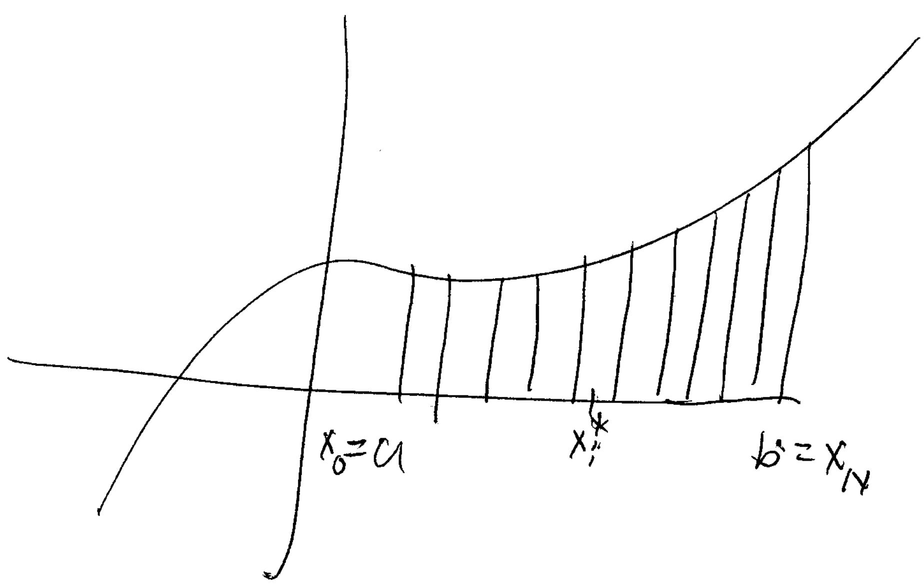
$\boxed{\text{Integration is getting the whole as the sum of the parts}}$

To compute a bulk quantity

1) Slice the problem into little pieces
2) Estimate the contribution of each piece.
3) Add them up.
4) Take a limit as you slice finer and finer.

Ex 1: Area under curve



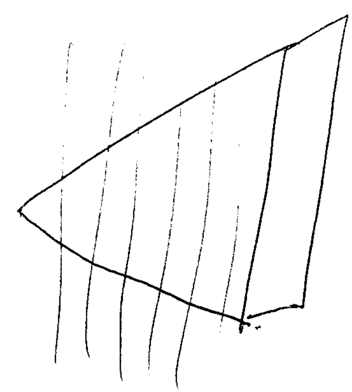
Pieces are ribbons $\simeq$ rectangles with area $f(x_i^*)\Delta x$

$$\text{Area under curve} \approx \sum_{i=1}^{N} f(x_i^*)\Delta x$$

$$\text{Area under curve} = \lim_{N\to\infty} \sum_{i=1}^{N} f(x_i^*)\Delta x$$

$$\stackrel{\text{def}}{=} \int_a^b f(x)\,dx$$

Ex 2: Volume of pyramid



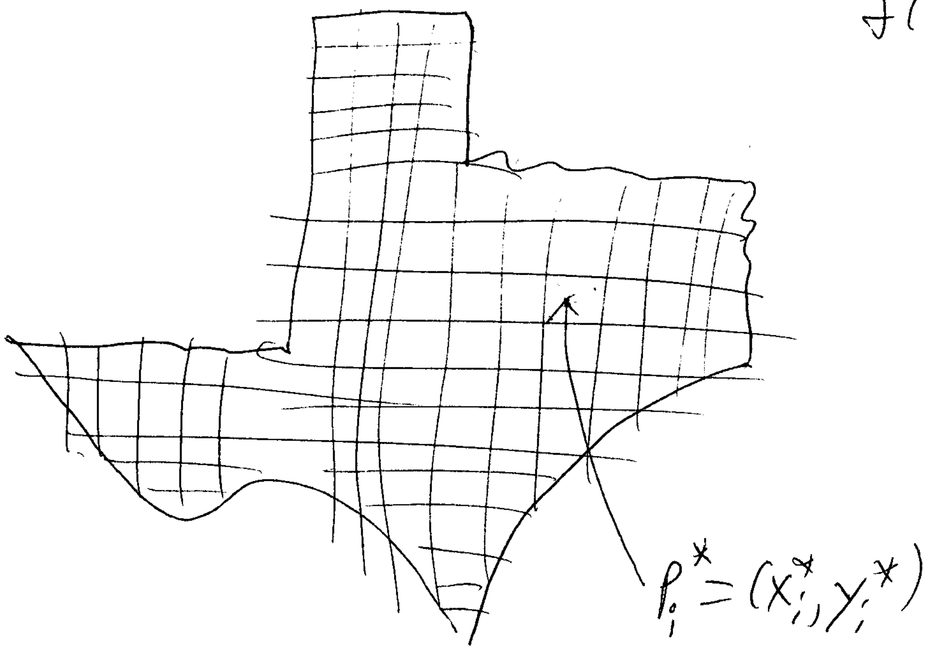
pieces are slabs of pyramid

Volume of each piece $\approx$ area $\times$ thickness

$$= x^2 \Delta x$$

$$\text{Volume} = \lim \sum x_i^{*2} \Delta x \stackrel{\text{def}}{=} \int_0^1 x^2\,dx$$

How much rain has fallen in TX since Jan 1?



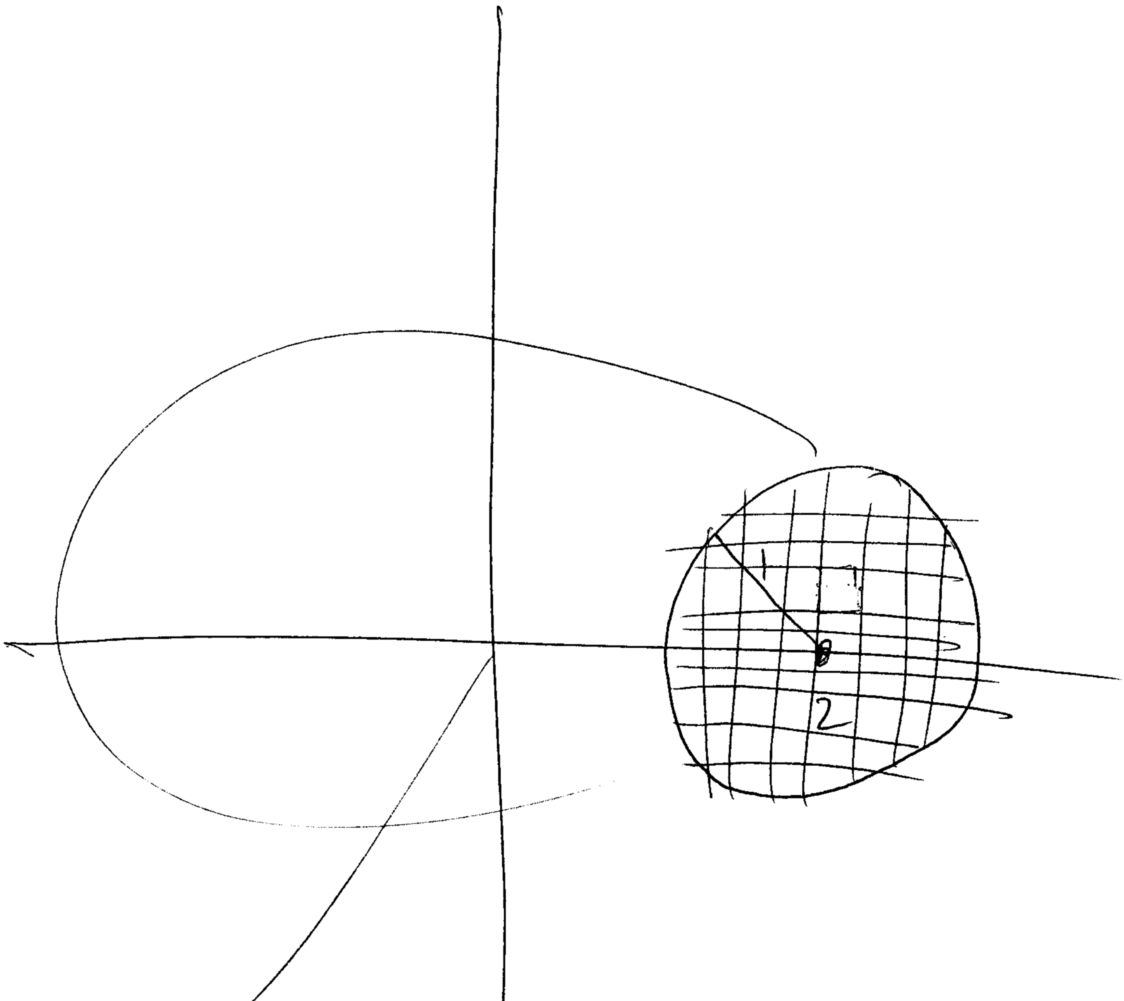
$f(x,y)$ = inches of rain at location (x,y)

rain in i-th piece $\approx$ $f(x_i^*, y_i^*) \cdot$ Area of pieces.

Total rainfall $\approx \sum_i f(p_i^*) \Delta area$

Total rainfall $= \lim_{N \to \infty} \sum_i f(p_i^*) \Delta area$

$$= \iint_{TX} f(x,y)\,dx\,dy$$



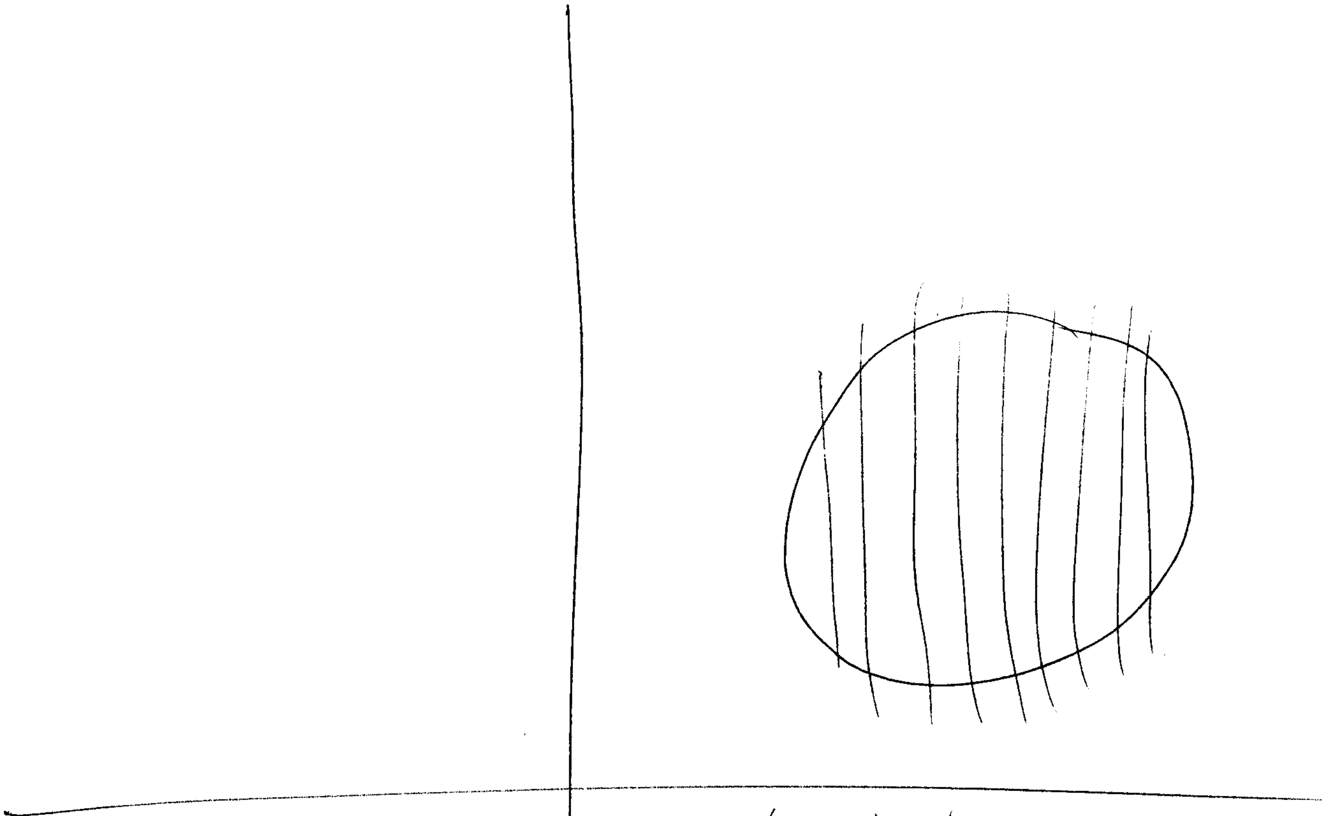
Rotate ~~circle~~ disk to get solid donut.

What is volume?

Each square sweeps out a bracelet of volume $2\pi x_i^* \Delta \text{area}$

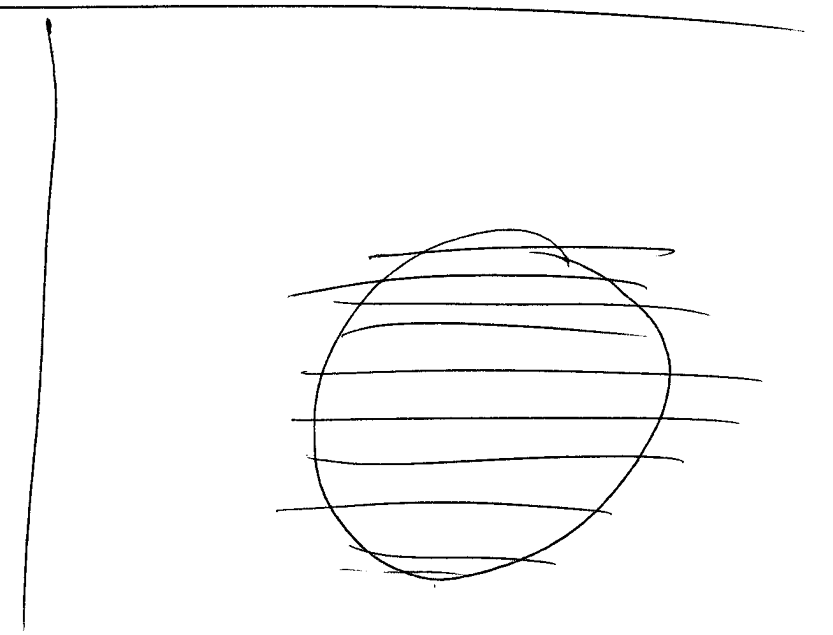
$$\text{Volume of donut} = \iint_{\text{Disk}} 2\pi x \, dx\, dy$$

$$\underline{\underline{\text{def}}} \quad \lim_{N \to \infty} \sum_{i=1}^{N} 2\pi x_i^* (\Delta \text{area})$$



each strip gives
cylidrical shell.
Compute volume of each shell
add them up.
Take a limit,

Same idea, only
with washers.



If f is a function of x,

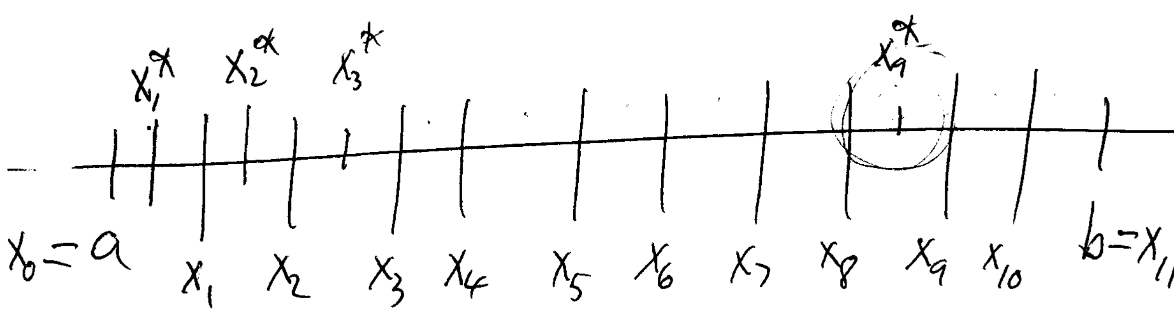
$$\int_a^b f(x)\,dx \stackrel{\text{def}}{=} \lim_{N\to\infty} \sum_{i=1}^{N} f(x_i^*)\,\Delta x$$

where x_i^* is a sample point between

$x_{i-1} = a + (i-1)\Delta x$ and

$x_i = a + i\,\Delta x$ and $\Delta x = \frac{b-a}{N}$.

Definite integral of $f(x)$ from $x=a$ to $x=b$



$N=11$

$\Delta x = \frac{b-a}{N} = \frac{b-a}{11}$

$x_1 = a + \Delta x$

$x_2 = x_1 + \Delta x = a + 2\Delta x$

$x_3 = x_2 + \Delta x = a + 3\Delta x$

$x_i = a + i\Delta x$

$x_i^* =$ sample pt in i-th interval between x_{i-1} and x_i

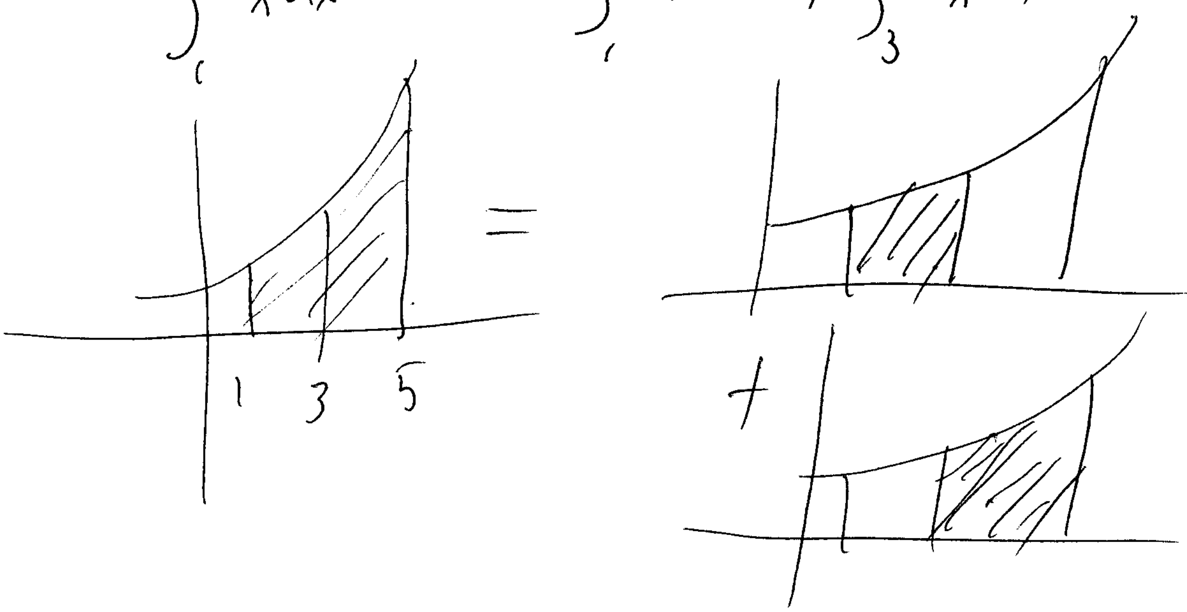
$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\boxed{\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx}$$

Ex: $$\int_1^5 x^2\,dx + \int_5^3 x^2\,dx = \int_1^3 x^2\,dx$$

$$\int_1^5 x^2\,dx = \int_1^3 x^2\,dx + \int_3^5 x^2\,dx$$



$$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\lim_{N\to\infty} \sum_{i=1}^{N} \cancel{f(x_i^*)}(f+g)(x_i^*)\,\Delta x$$

$$= \lim_{N\to\infty} \sum_{i=1}^{N} \left(f(x_i^*)+g(x_i^*)\right)\Delta x$$

$$= \lim_{N\to\infty} \left(\sum f(x_i^*)\Delta x + \sum g(x_i^*)\Delta x\right)$$

$$= \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

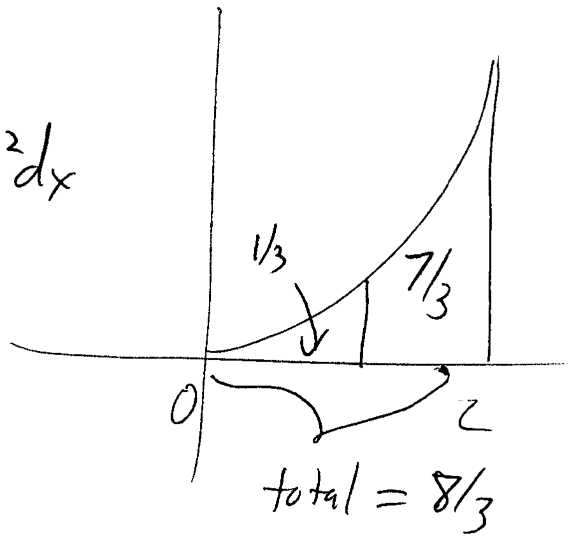
$$\int_a^b c\, f(x)\,dx = c \int_a^b f(x)\,dx$$

$$\int_0^1 x^2\,dx = \frac{1}{3} \qquad \int_0^2 x^2\,dx = \frac{8}{3}$$

$$\int_1^2 x^2\,dx = \int_1^0 x^2\,dx + \int_0^2 x^2\,dx$$

$$= -\int_0^1 x^2\,dx + \int_0^2 x^2\,dx$$

$$= -\frac{1}{3} + \frac{8}{3} = \frac{7}{3}$$



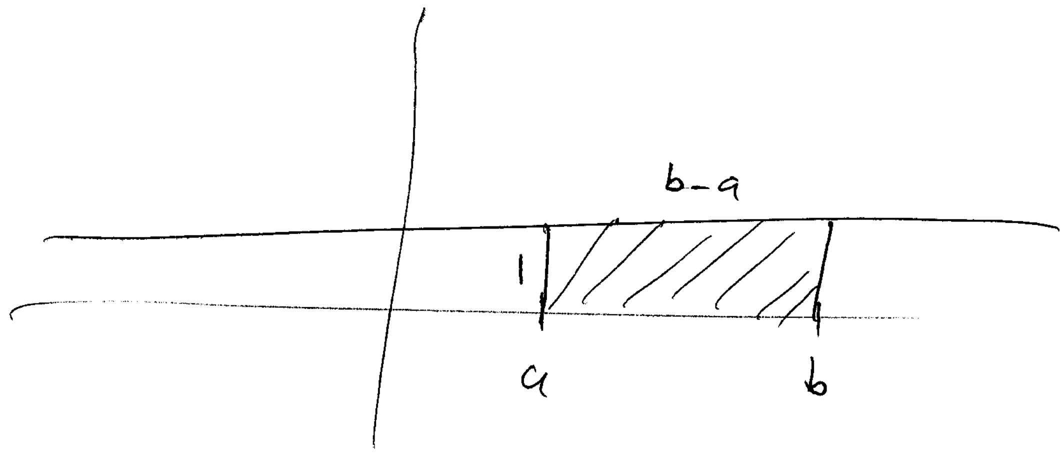
$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(y)\,dy = \int_a^b f(s)\,ds$$

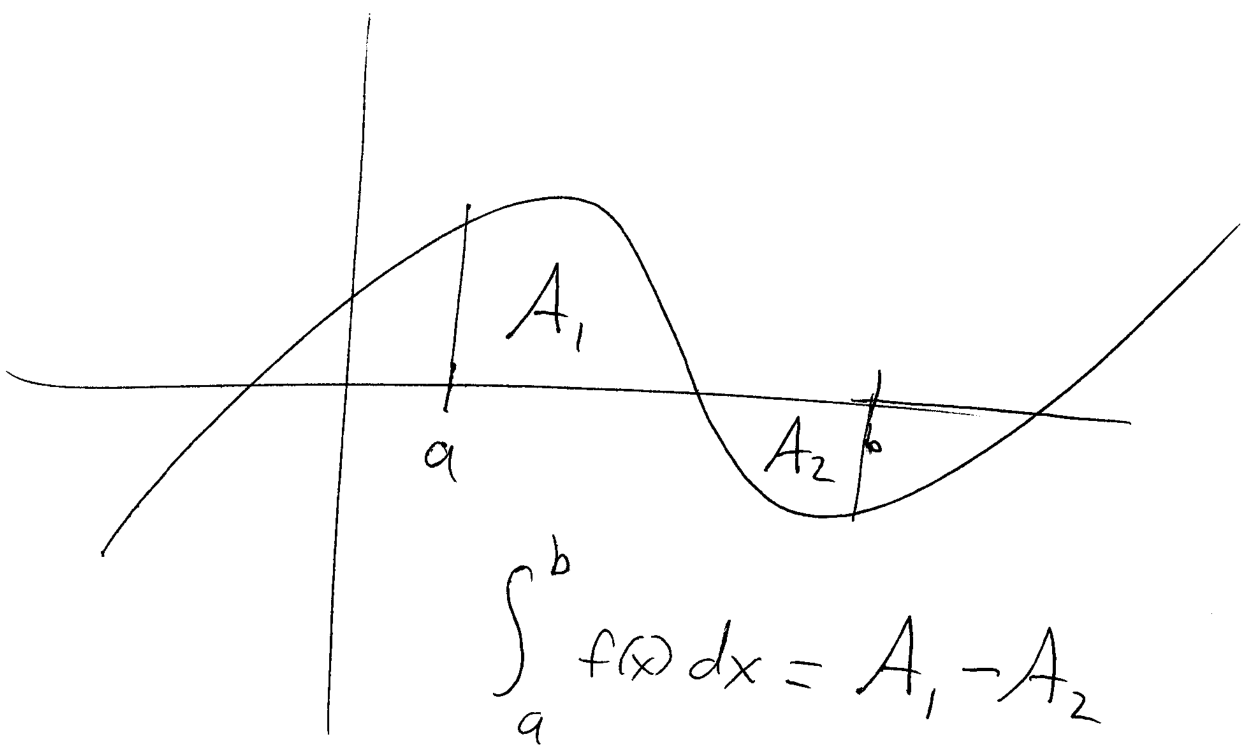
etc.

$$= \lim_{N\to\infty} \sum_{i=1}^{N} f\left(\begin{array}{c}\text{some \# between} \\ a+(i-1)\left(\frac{b-a}{N}\right) \text{ and} \\ a+i\left(\frac{b-a}{N}\right)\end{array}\right) \cdot \frac{b-a}{N}$$

$$\sum_{i=1}^{15} (i^2-7) \quad \text{or} \quad \sum_{j=1}^{15} (j^2-7)$$

$$\int_a^b 1\,dx = b-a$$

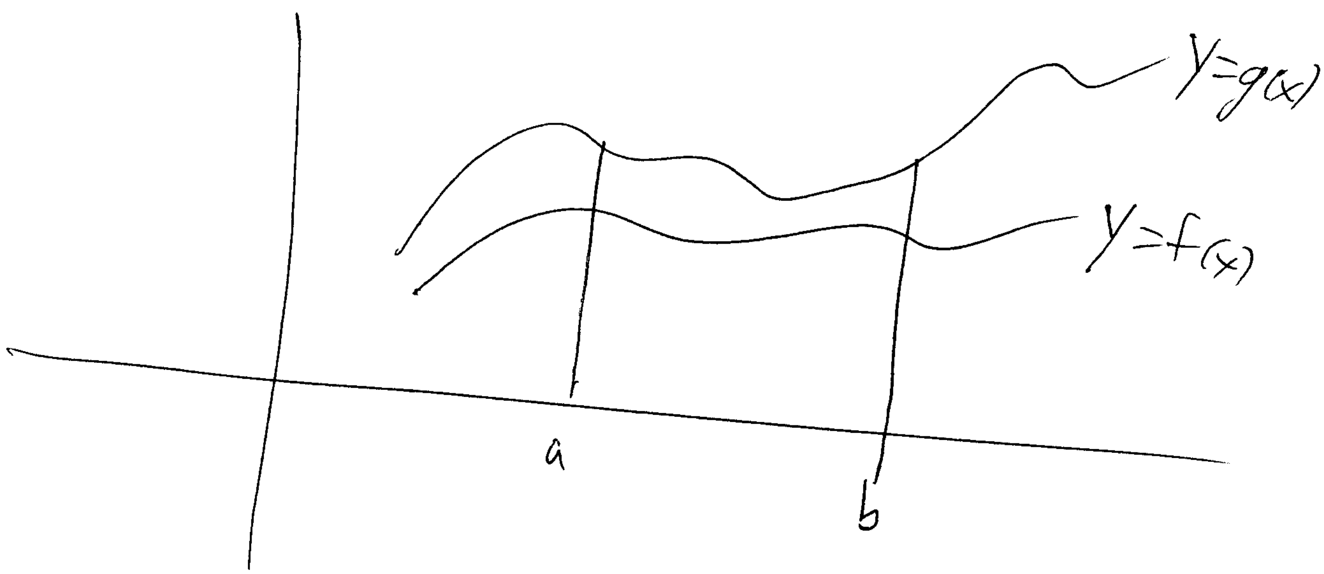




$$\int_a^b f(x)\,dx = A_1 - A_2$$

If $f(x) \leq g(x)$ and $b \geq a$, then

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$



$f(x_i^*) \leq g(x_i^*)$, so

$$\sum_{i=1}^{N} f(x_i^*)\Delta x \leq \sum_{i=1}^{N} g(x_i^*)\Delta x$$

$$\lim_{N\to\infty} \sum_{i=1}^{N} f(x_i^*)\Delta x \leq \lim_{N\to\infty} \sum_{i=1}^{N} g(x_i^*)\Delta x$$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx.$$

$$\int_a^b f(x)\,dx = \lim_{N\to\infty} \sum_{i=1}^{N} f(x_i^*)\,\Delta x \qquad \boxed{\text{Defn}}$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$$\int_a^b \left(f(x) \pm g(x)\right) dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b 1\,dx = (b-a)$$

If $f(x) \le g(x)$ and $a \le b$,

$$\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$$

$\int_a^b f(x)\,dx =$ Area under the curve $-$ Area over the curve.

$$\boxed{\text{Usefull summation formulas}}$$

$$\sum_{i=1}^{N} i^0 = N$$

$$\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$$

$$\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\sum_{i=1}^{N} i^3 = \left(\frac{N(N+1)}{2}\right)^2$$