$$\lim_{x \to \cancel{a}\,\infty} g(x) = L$$

Suppose $f(x)$ is continuous at L.

Then $\lim_{x \to \cancel{a}\,\infty} f(g(x)) = f(L)$.

$y = g(x)$

Whenever $x \approx a$, $g(x) \approx L$

$$\lim_{y \to L} f(y) = f(L)$$

When $y \approx L$ $f(y) \approx f(L)$

When $\cancel{x \approx a}$ x large, $y \approx L$

when $y \approx L$, $f(y) \approx f(L)$

When $\cancel{x \approx a}$ x large, $f(g(x)) \approx f(L)$

$$\lim_{\substack{\cancel{x \to a} \\ x \to \infty}} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

$$\lim_{x\to 1}\sqrt{\frac{x^2-1}{x-1}}$$

$$\lim_{x\to 1}\frac{x^2-1}{x-1}=2$$

$$=\sqrt{\lim_{x\to 1}\frac{x^2-1}{x-1}}=\sqrt{2}$$

$$\lim_{x\to 1}\ln\left(\frac{x^2-1}{x-1}\right)=\ln\left(\lim_{x\to 1}\frac{x^2-1}{x-1}\right)=\ln(2)$$

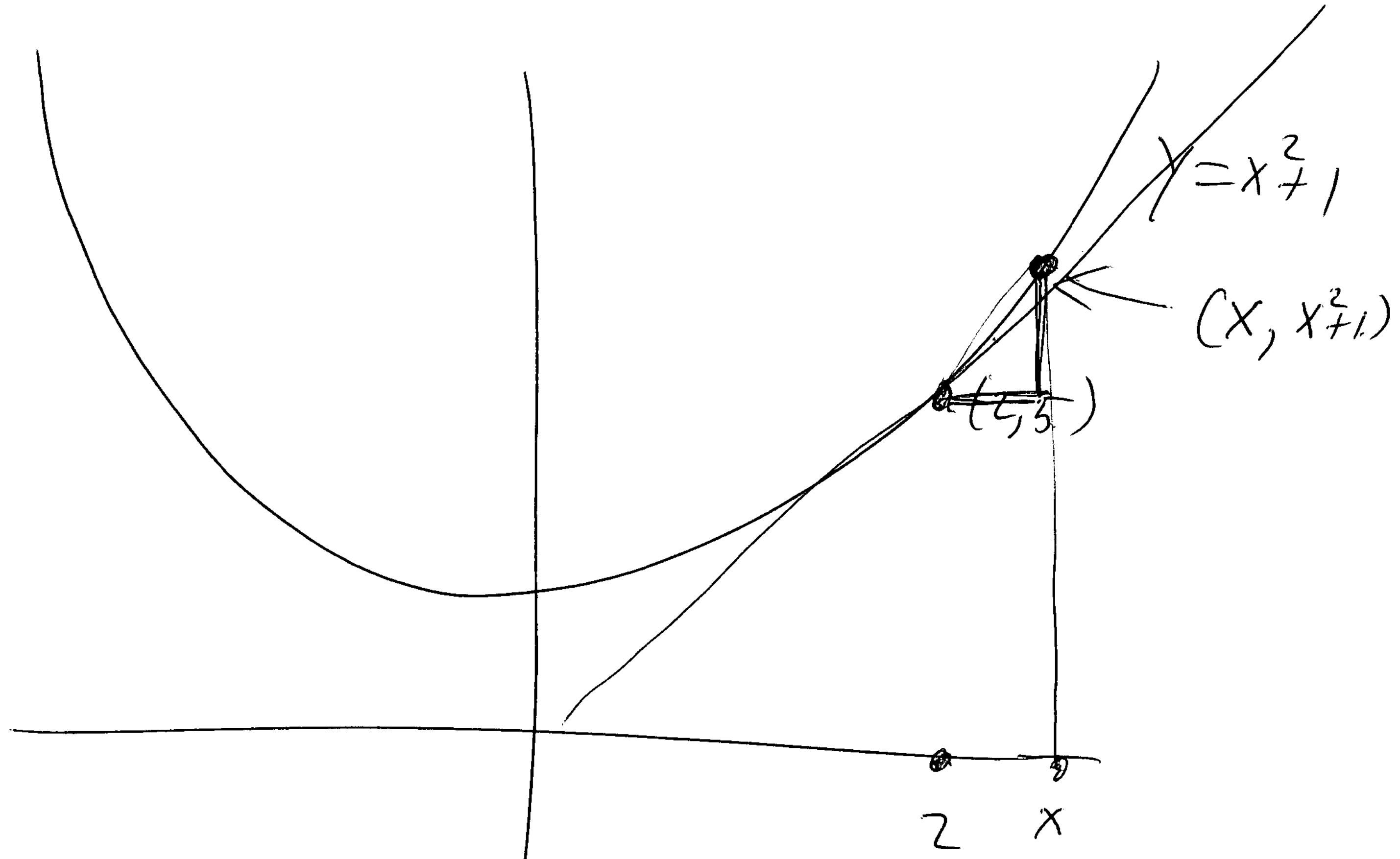
$$\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)^5=\left(\lim_{x\to 1}\frac{x^2-1}{x-1}\right)^5=2^5=32$$

$$\lim_{x\to\infty}\sqrt{\frac{4x^3+2x+1}{x^3-7}}=\sqrt{\lim_{x\to\infty}\frac{4x^3+2x+1}{x^3-7}}=\sqrt{4}=2$$

$$\lim_{x\to 1} g(x)=0$$

$\sqrt{x}$ not continuous at $x=0$.

$$\lim_{x\to 1}\sqrt{g(x)}=???$$



Rise $= y_2 - y_1 = x^2 + 1 - 5 = \Delta y$

Run $= x - 2 = \Delta x$

Slope of secant line $= \dfrac{\Delta y}{\Delta x} = \dfrac{x^2-4}{x-2}$

Slope of tangent line $= \lim\limits_{\Delta x \to 0}$ (slope of secant line)

$$= \lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2} (x+2) = 4$$

Tangent line goes through $(2,5)$

slope $= 4$

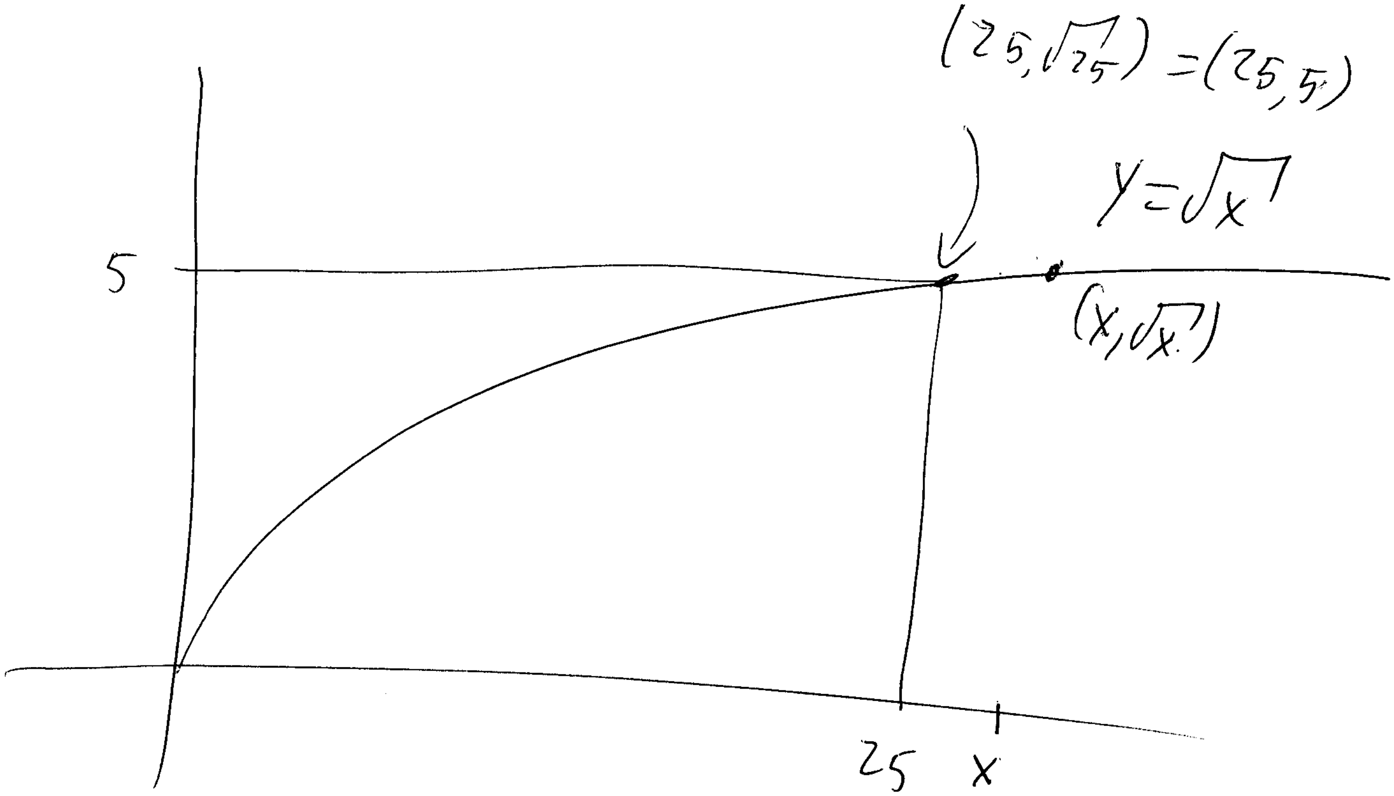
$$y - 5 = 4(x-2)$$

$$y = 4x - 3$$ equation of tangent line

What is $f(2.07) \approx 4(2.07) - 3$

$= 5.28$

$f(2.07) = (2.07)^2 + 1 = 5.2849$



Rise $= \sqrt{x} - \sqrt{25} = \sqrt{x} - 5$

Run $= x - 25$

Slope of secant $= \dfrac{\sqrt{x} - 5}{x - 25}$

Slope of tangent $= \lim\limits_{x \to 25} \dfrac{\sqrt{x} - 5}{x - 25}$

$$= \lim_{x \to 25} \frac{(\sqrt{x} - 5)(\sqrt{x} + 5)}{(x - 25)(\sqrt{x} + 5)} = \lim_{x \to 25} \frac{\cancel{(x - 25)}}{\cancel{(x - 25)}(\sqrt{x} + 5)}$$

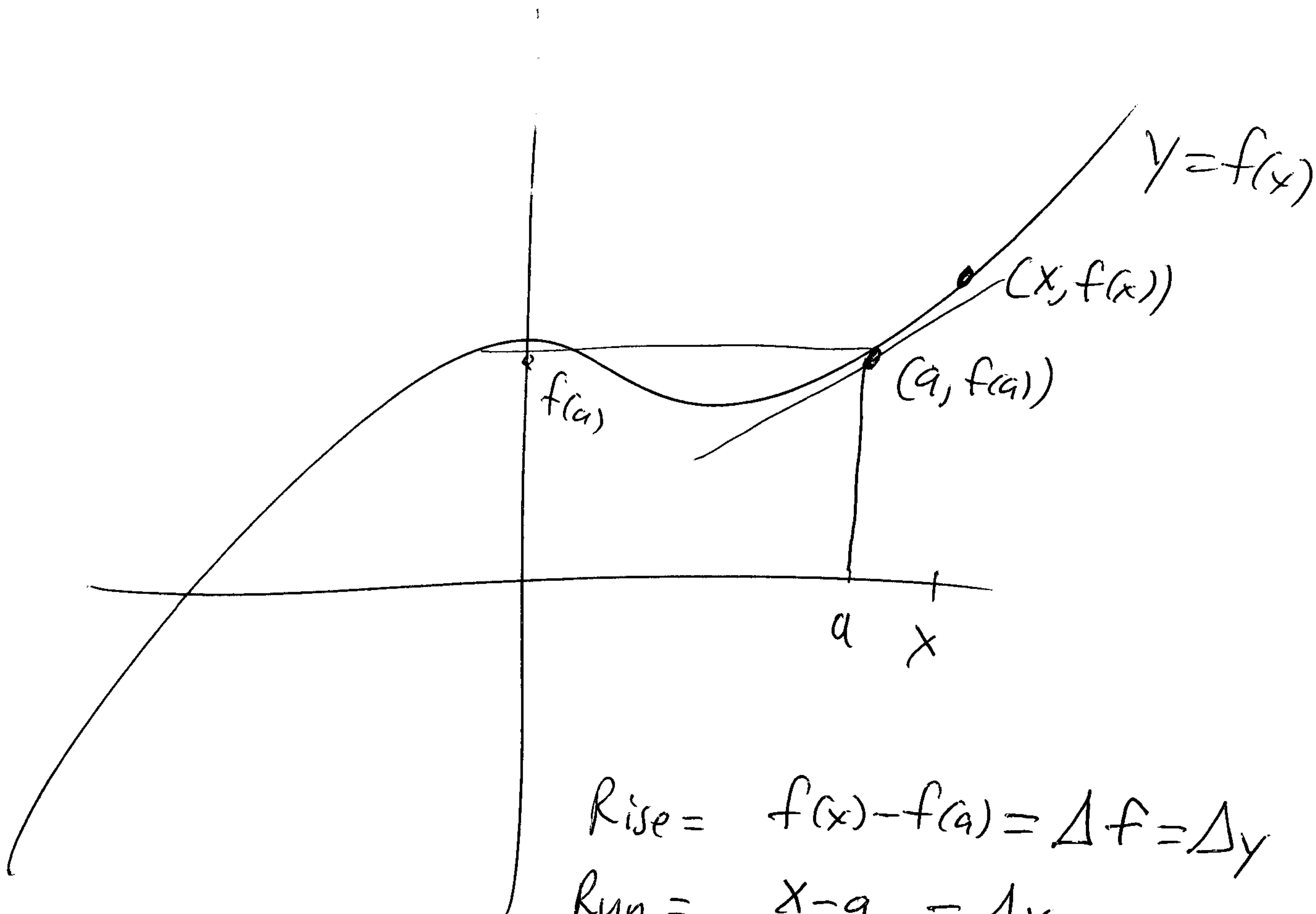
$$= \lim_{x \to 25} \frac{1}{\sqrt{x} + 5} = \frac{1}{10}$$

$y - 5 = .1(x - 25)$ Equation of tangent line.

$y = .1x + 2.5$

$\sqrt{25.1} \approx 5.01$

$\sqrt{25.1} = 5.00995\ldots$



Rise $= f(x)-f(a) = \Delta f = \Delta y$

Run $= x-a = \Delta x$

Slope of secant line $= \dfrac{\Delta y}{\Delta x} = \dfrac{f(x)-f(a)}{x-a}$

Slope of tangent line $= \lim\limits_{x\to a}\left(\dfrac{f(x)-f(a)}{x-a}\right)$

$=$ Derivative of f at $x=a$.

$= f'(a) = \left.\dfrac{dy}{dx}\right|_{x=a} = \left.\dfrac{df}{dx}\right|_{x=a}$

Derivative of x^2+1 at $x=2$ was 4

Derivative of $\sqrt{x}$ at $x=25$ was $\frac{1}{10}$

$$f'(a) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} \qquad \begin{pmatrix} h = x-a \\ x = a+h \end{pmatrix}$$

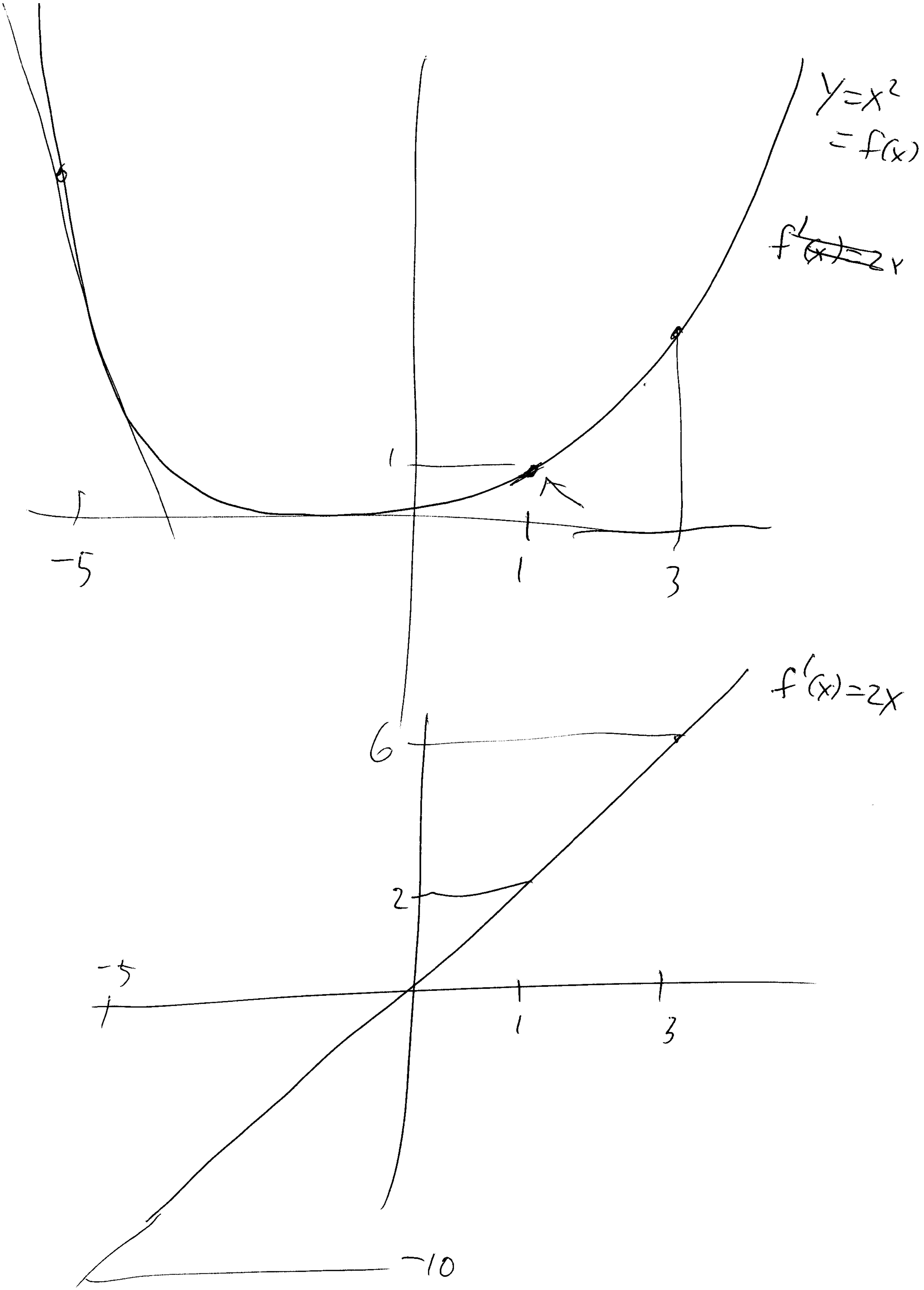
$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$f(x)=x^2$

$$f(x+h) = (x+h)^2 = x^2+2xh+h^2$$

$$f(x+h)-f(x) = 2xh+h^2$$

$$f'(x) = \lim_{h \to 0} \frac{2xh+h^2}{h} = \lim_{h \to 0} (2x+h) = 2x$$



y=x^2
=f(x)
~~f'(x)=2x~~
1
-5
1
3
f'(x)=2x
6
2
-5
1
3
-10

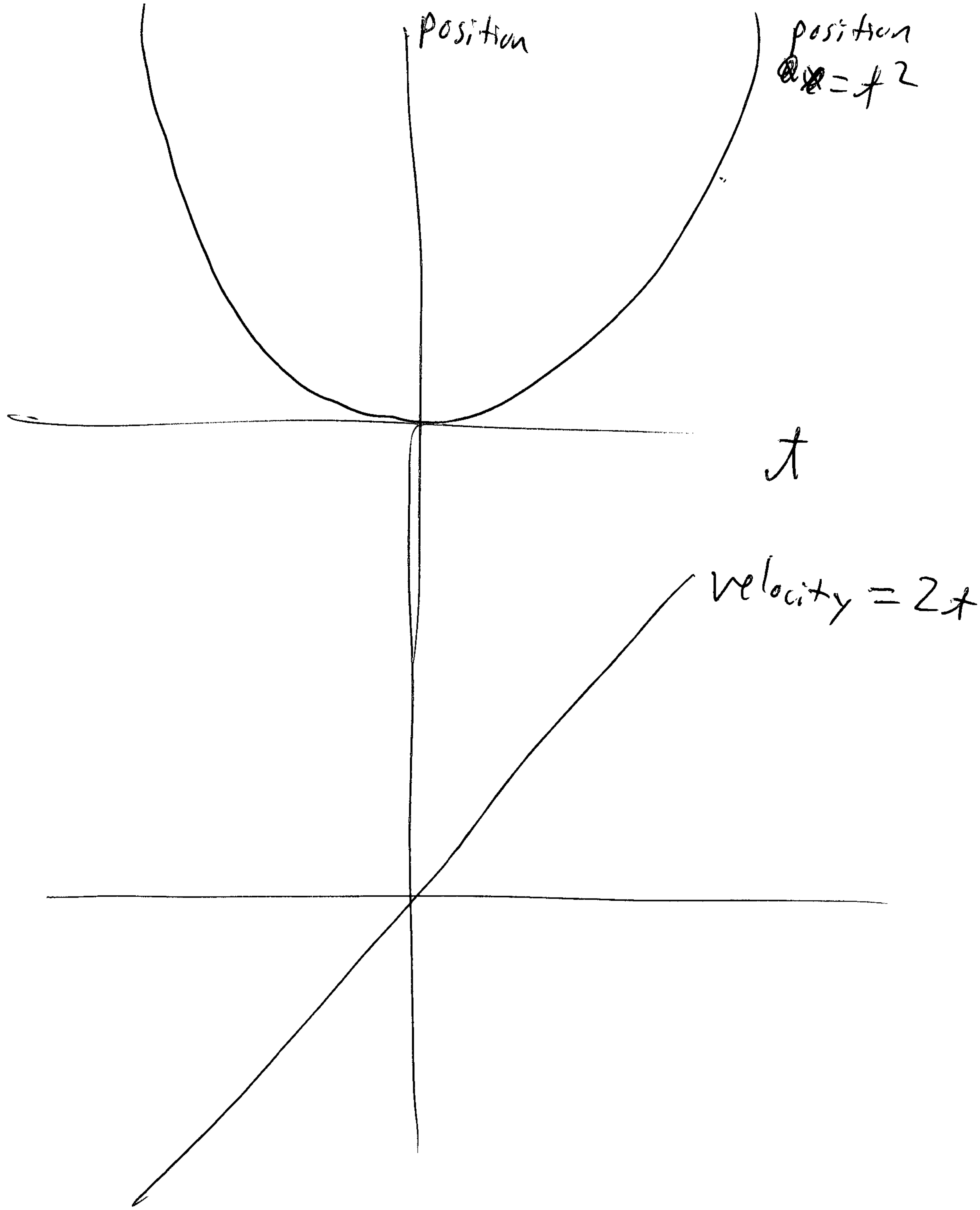
Suppose position $= f(t)$

Velocity at time a $= \lim_{t \to a} \dfrac{f(t) - f(a)}{t - a}$

$= \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ $\quad h = t-a$

velocity at ~~timed~~ time t $= \lim_{h \to 0} \dfrac{f(t+h) - f(t)}{h}$

Velocity = derivative of position.


position
position
$x = t^2$
t
velocity $= 2t$

Derivative = rate of change of a function.

$$= \lim_{(\Delta \text{input} \to 0)} \frac{\Delta \text{ output}}{\Delta \text{ input}}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \left(= \frac{df}{dx} = \frac{dy}{dx} \right)$$

$$f(x) = \sqrt{x}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \lim_{h \to 0} \frac{(\cancel{x}+\cancel{h}) \cancel{-x}}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$