Derivative = rate of change.

= slope of tangent line.

= sensitivity = $\dfrac{\text{Change in output}}{\text{Change in input}}$

= velocity.

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = f'(a)$$

$$= \frac{df}{dx}(a) = \ldots$$

$$= dy/dx$$

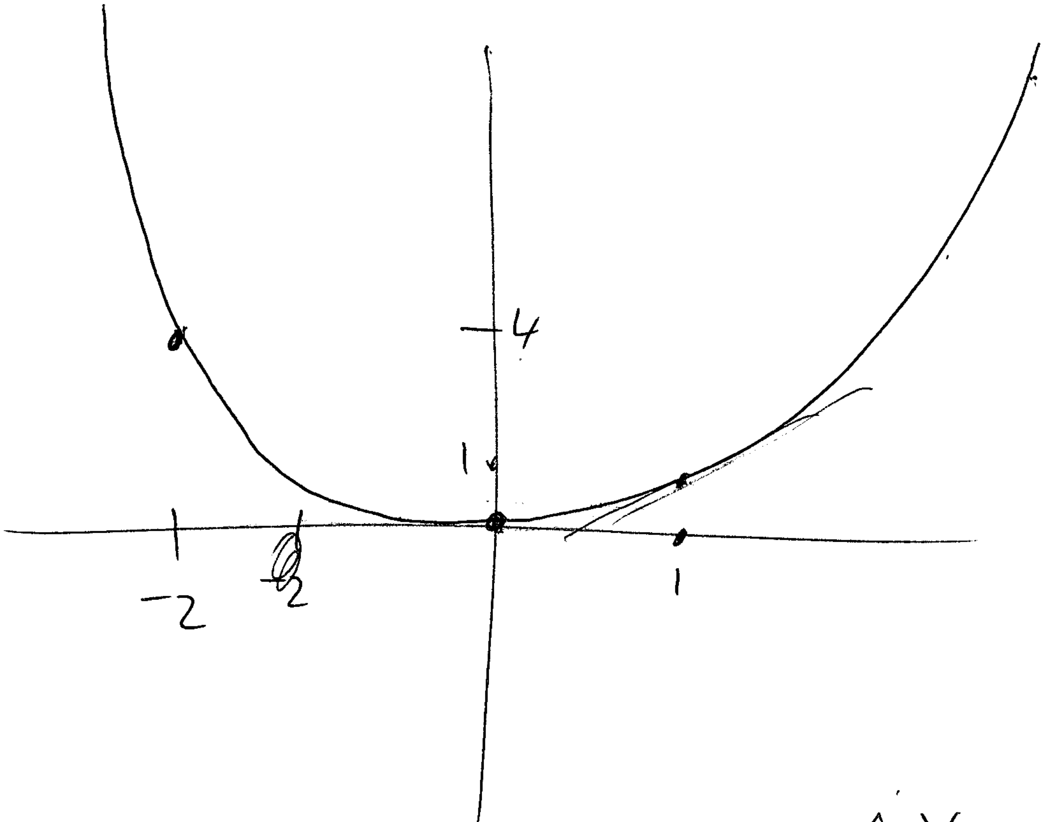
If $f(x) = x^2$.

$$f'(2) = \lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} (x+2) = 4$$

$$f'(a) = \lim_{x \to a} \frac{x^2-a^2}{x-a} = \lim_{x \to a} (x+a) = 2a$$

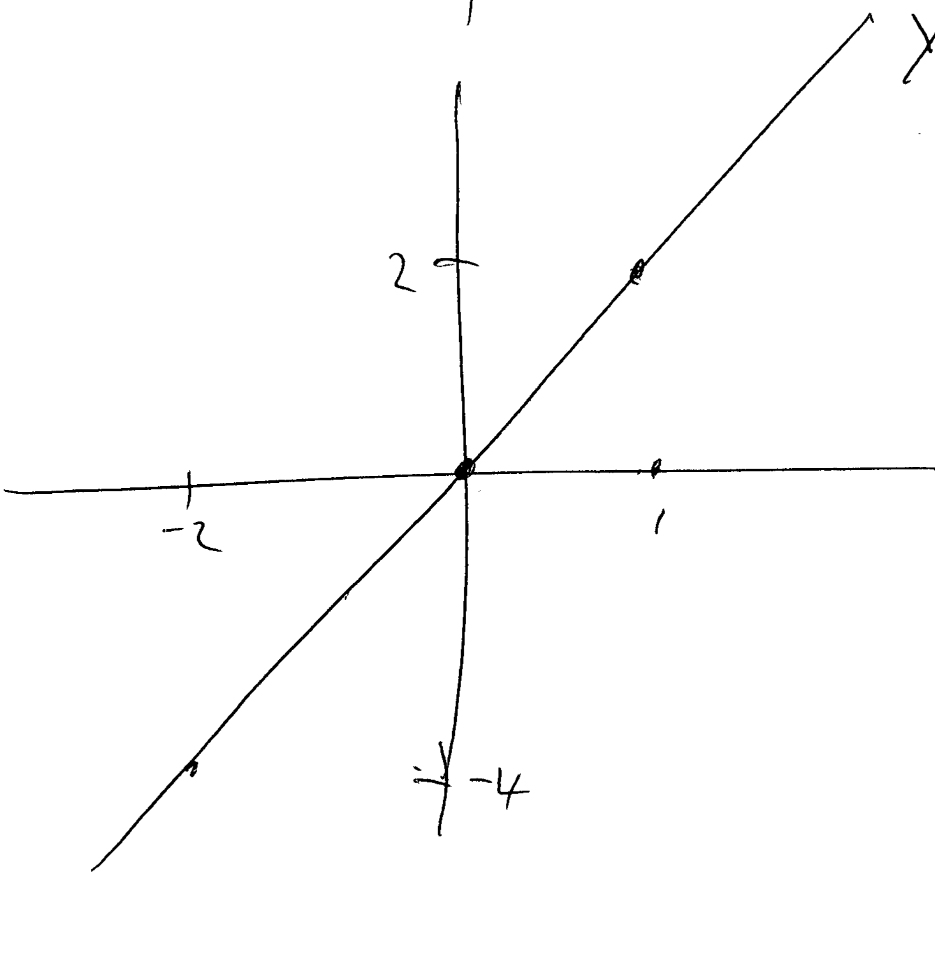
$$f'(x) = 2x$$

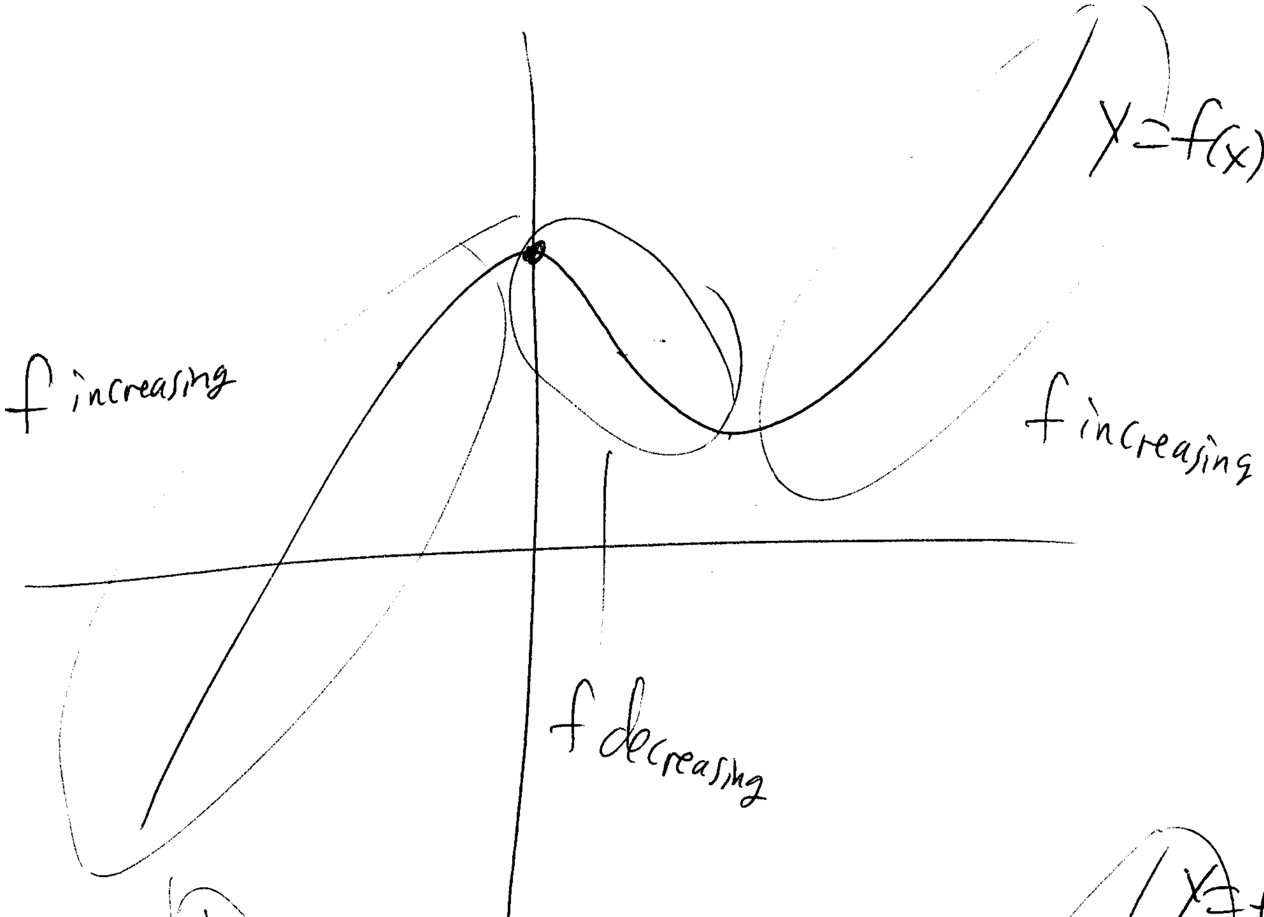
$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

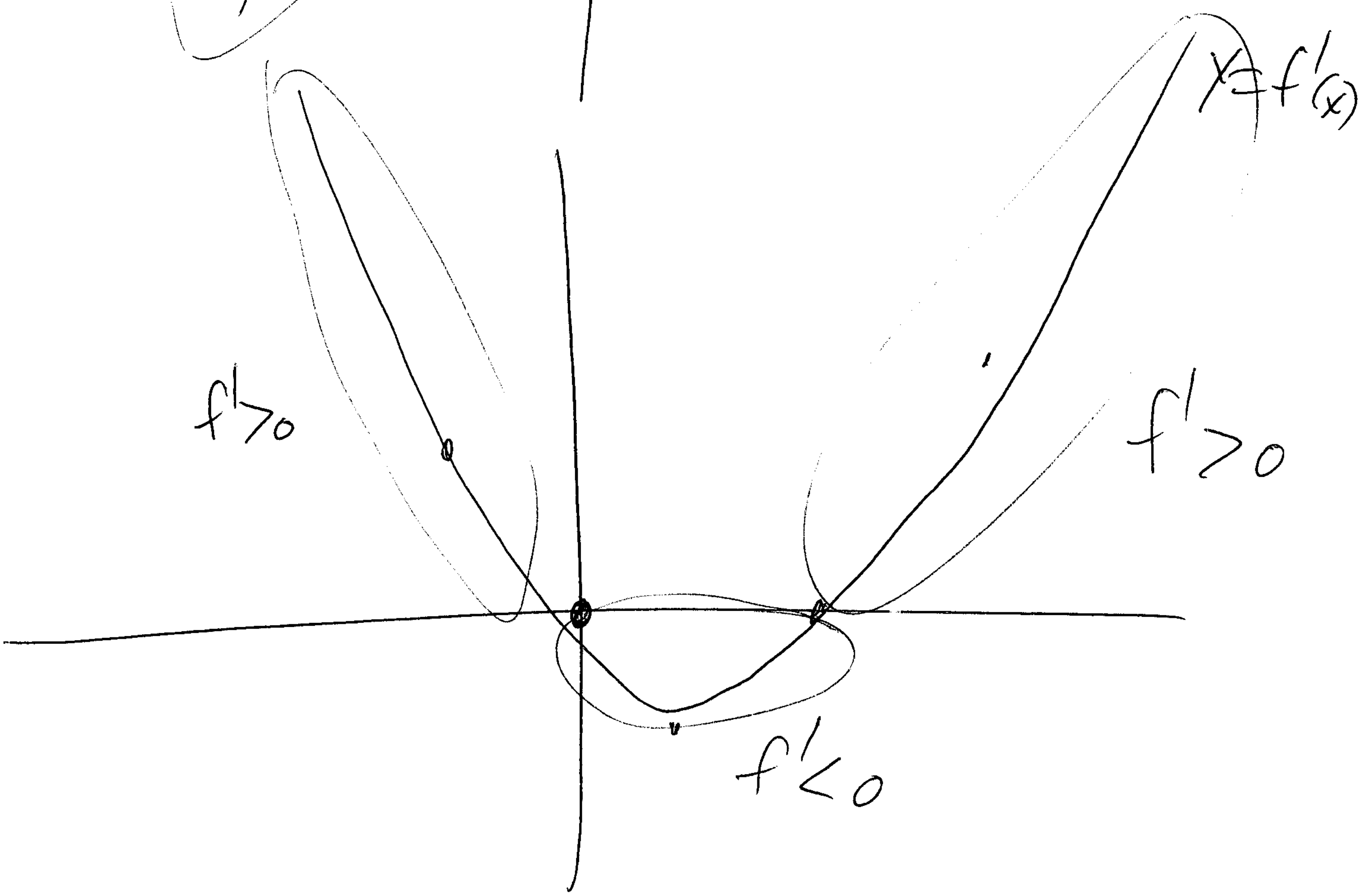


$y = x^2$
$= f(x)$

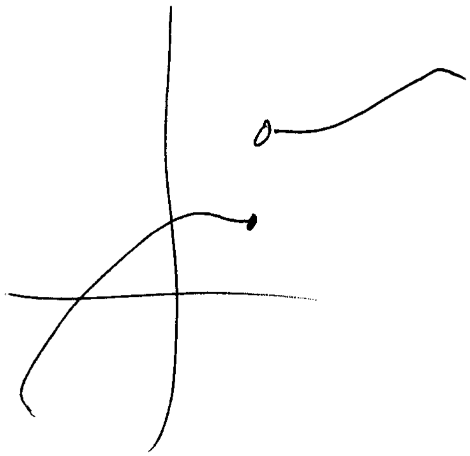
$y = 2x = f'(x)$

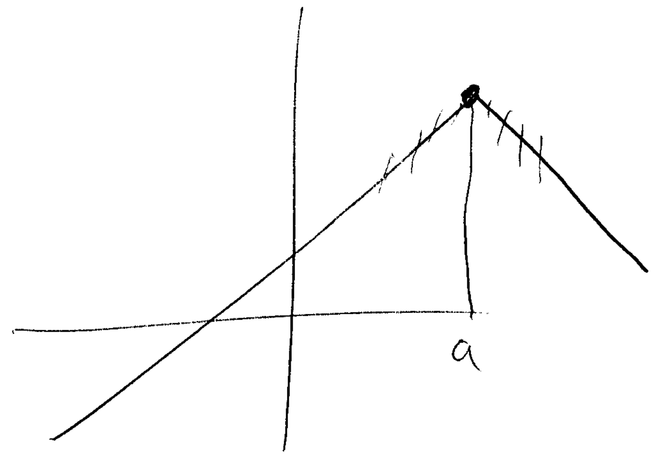


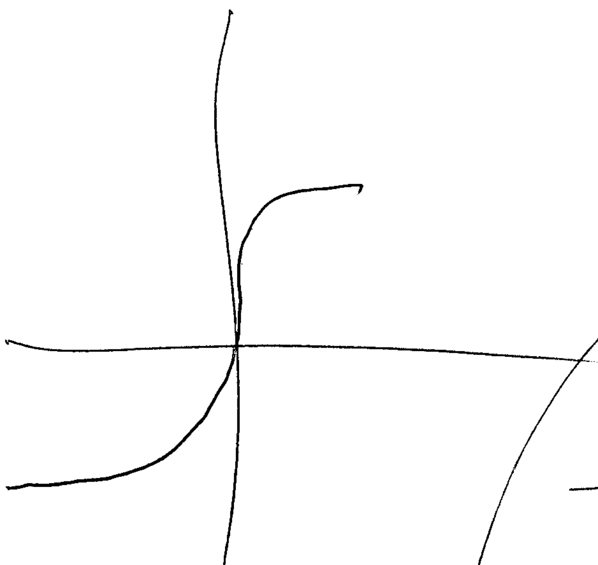

Y=f(x)
f increasing
f increasing
f decreasing

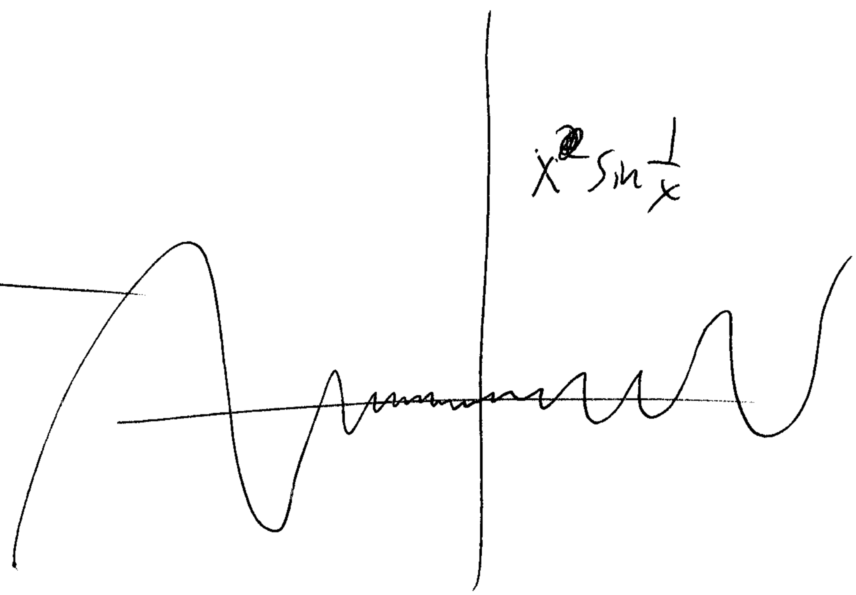

Y=f'(x)
f'>0
f'>0
f'<0

What can go wrong?









$f''(x) =$ derivative of $f'(x)$

$f'''(x) =$ derivative of $f''(x)$

$f^{(17)}(x) =$ 17-th derivative of $f(x)$

$$= \frac{d^{17}f}{dx^{17}}$$

If $f'(a)$ ~~existes~~ exists = f is <u>differentiable</u> at a,

<u>Thm</u> If $f'(a)$ exists, f is continuous at a.

$$\lim_{x\to a} f(x) = f(a) + \lim_{x\to a} \big(f(x) - f(a)\big)$$

$$= f(a) + \lim_{x\to a} \frac{f(x)-f(a)}{(x-a)} (x-a)$$

$$= f(a) + \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \lim_{x\to a} (x-a)$$

$$= f(a) + f'(a) \cdot 0 = f(a)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

---

1) If $f(x) = c$, $f'(x) = 0$.

Pf $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0$.

---

2) $\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d\,f(x)}{dx} + \frac{d\,g(x)}{dx}$

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - \left(f(x) + g(x)\right)}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right)$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x)$$

$$(f+g)' = f' + g'$$

3) $(f-g)' = f'-g'$

---

4) If $n =$ ~~positive~~ non-negative integer, $\frac{d}{dx} x^n = n x^{n-1}$

$f(x) = x^n$

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a} \frac{x^n - a^n}{x-a}$$

$$= \lim_{x \to a} \left(x^{n-1} + a x^{n-2} + a^2 x^{n-3} + \cdots + a^{n-1}\right)$$

$$= a^{n-1} + a^{n-1} + \cdots + a^{n-1} = n a^{n-1}$$

$f'(x) = n x^{n-1}$

$$\frac{d}{dx}\left(2x^3 + 5x^2 - 7x + 19\right) = \frac{d}{dx}(2x^3) + \frac{d}{dx}(5x^2) - \frac{d}{dx}(7x) + \frac{d}{dx}(19)$$

$$= 2\frac{d}{dx}(x^3) + 5\frac{d}{dx}(x^2) - 7\frac{d}{dx}x + \frac{d}{dx}19$$

$$= 2(3x^2) + 5(2x) - 7 \cdot 1 + 0 = 6x^2 + 10x - 7$$

5) $\frac{d}{dx}(cf) = c\,\frac{df}{dx}$

$f(x) = 2^x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2^{x+h}-2^x}{h}$$

$$= \lim_{h\to 0} \frac{2^x(2^h-1)}{h}$$

$$= 2^x \lim_{h\to 0}\left(\frac{2^h-1}{h}\right) \approx .69\,2^x$$

$$\frac{d}{dx}e^x = e^x \boxed{\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)} = e^x$$

$\boxed{\text{NOT } xe^{x-1}}$

$$\frac{d}{dx}a^x = a^x \ln(a)$$

Product rule: $\quad (f(x)\cdot g(x))' = f(x)\,g'(x) + f'(x)\,g(x).$

$$\frac{d}{dx}\left(x^2e^x\right) = x^2\,(e^x)' + (x^2)'\,e^x$$

$$= x^2 e^x + 2x\,e^x = (x^2+2x)\,e^x$$

$$\frac{d}{dx}\left((x^3+2x+7)\cdot(3x+1)\right) = (x^3+2x+7)\cdot 3$$

$$+ (3x^2+2)\cdot(3x+1)$$

$$\frac{d}{dx}\left(\frac{f}{g}\right)$$