"$\lim_{x \to a} f(x) = L$" means

"when x is close to a (but $\neq a$), $f(x)$ is close to L"

If $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$, then

1) $\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L + M$

2) $\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3) $\lim_{x \to a} c\, f(x) = c \lim_{x \to a} f(x)$

4) $\lim_{x \to a} f(x) g(x) = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$

5) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}$ $\boxed{\text{if } \lim g(x) \neq 0}$

6) If ~~n~~ $n =$ positive integer,

$\lim (f(x))^n = (\lim f(x))^n$

7) $\lim_{x \to a} x = a$

$\lim_{x \to a} (x^2) = \left(\lim_{x \to a} (x)\right)^2 = a^2$

8) $\lim_{x \to a} x^n = a^n$

9) If $p(x)$ is a polynomial, $\lim_{x \to a} p(x) = p(a)$

10) $\lim_{x \to a} \sqrt{f(x)} = \sqrt{\lim f(x)}$ as long as $\lim f(x) > 0$

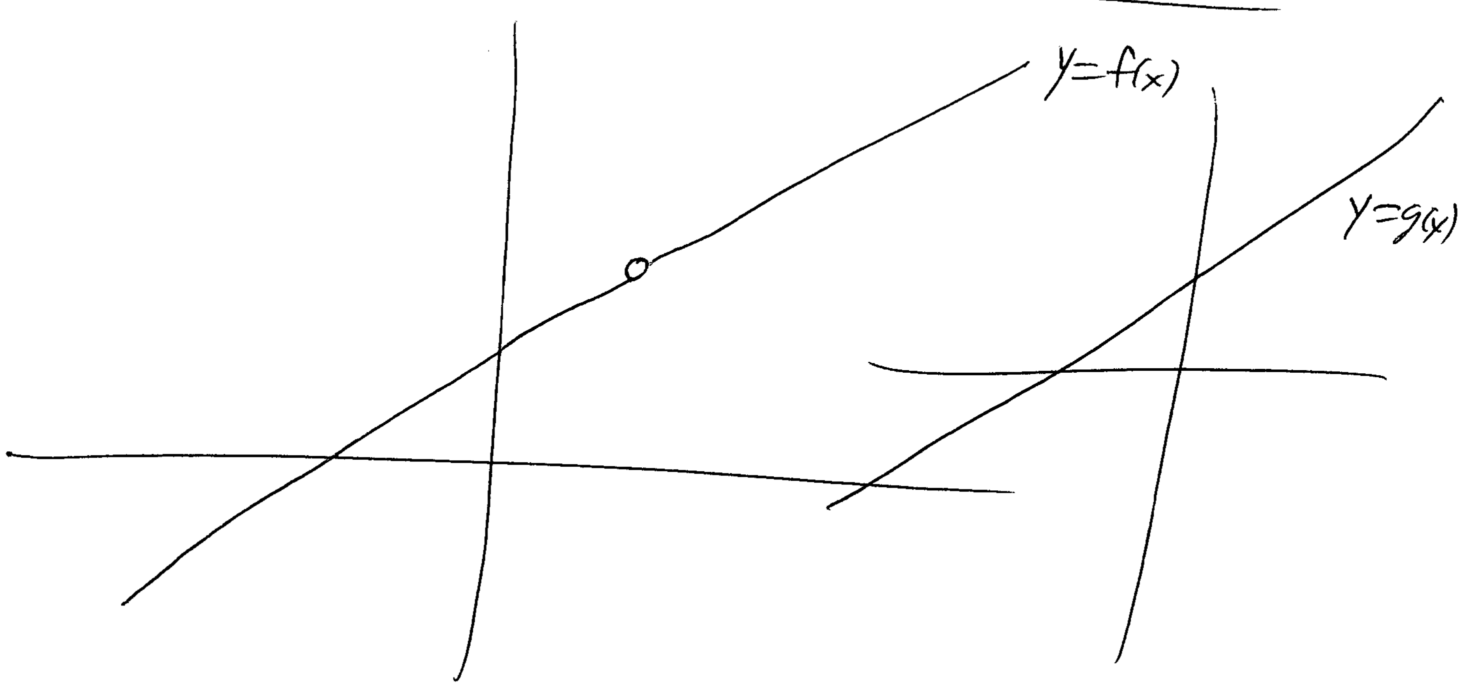
$$\lim_{x\to 1} \frac{x^2-1}{x+3} = \frac{\lim_{x\to 1}(x^2-1)}{\lim_{x\to 1}(x+3)} = \frac{0}{4} = 0$$

$$\lim_{x\to 1} \overset{f(x)}{\frac{x^2-1}{x-1}} = \frac{(x-1)(x+1)}{(x-1)}$$

$g(x) = x+1$

If $f(x) = g(x)$ whenever $x \neq a$,

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x).$$



$$\lim_{x \to 5} \frac{x^2-3x-10}{x^2-25} = \lim_{x \to 5} \frac{(x-5)(x+2)}{(x-5)(x+5)}$$

$$= \lim_{x \to 5} \frac{x+2}{x+5} = \frac{\lim_{x \to 5}(x+2)}{\lim_{x \to 5}(x+5)} = \frac{7}{10}$$

$$f(x) = \frac{x^2-3x-10}{x^2-25} \qquad g(x) = \frac{x+2}{x+5}$$

$$\lim_{x\to 1} \frac{16x^2-16}{x-1} = \lim_{x\to 1} \frac{16(x-1)(x+1)}{(x-1)}$$

$$= \lim_{x\to 1} 16(x+1) = 32$$

$$\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$$

$$(a+b)(a-b) = a^2-b^2$$

$$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$$

$$= \lim_{x\to 0} \frac{(\sqrt{x^2+9}-3)(\sqrt{x^2+9}+3)}{x^2(\sqrt{x^2+9}+3)}$$

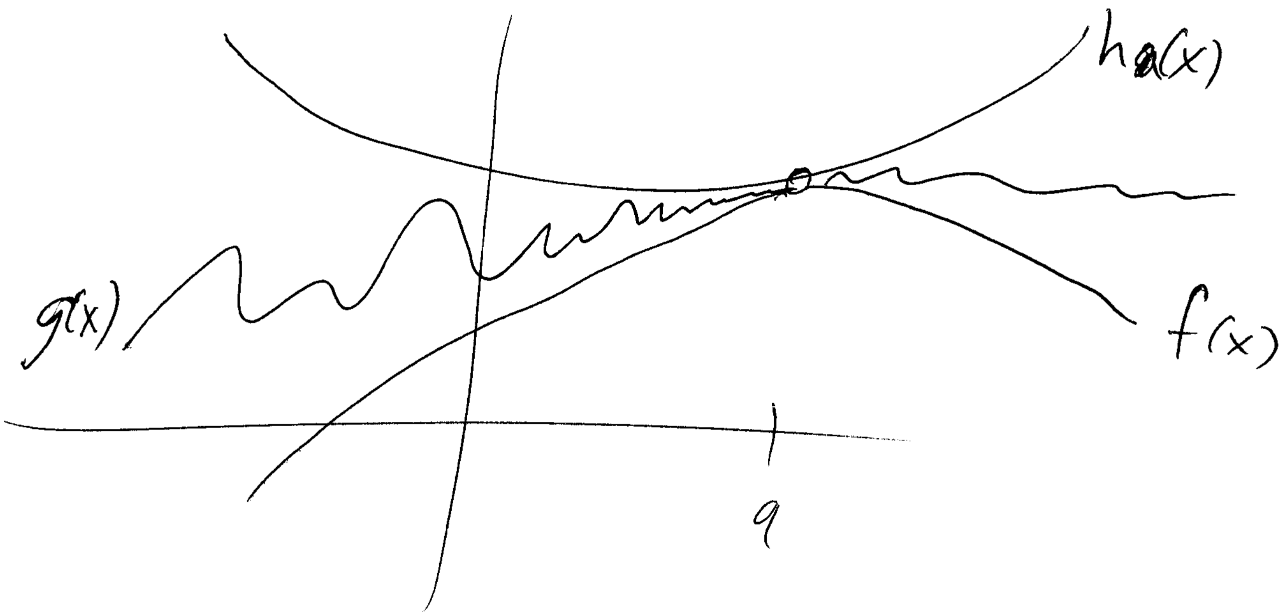
$$= \lim_{x\to 0} \frac{x^2+9-9}{x^2(\sqrt{x^2+9}+3)} = \lim_{x\to 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+9}+3} = \frac{1}{6}$$

Sandwich (aka Squeeze) Thm

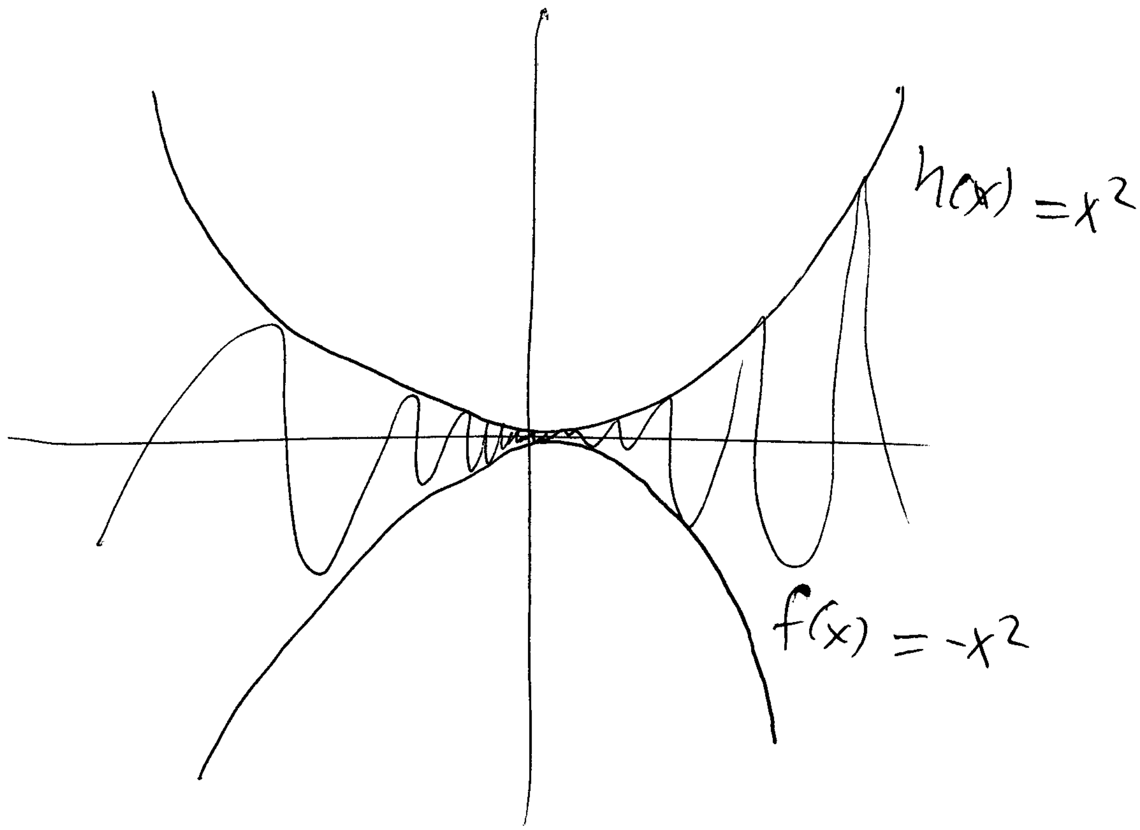
If $f(x) \le g(x) \le h(x)$ for x close to (but not $=$) a, ~~the~~ and if $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then

$$\lim_{x\to a} g(x) = L.$$

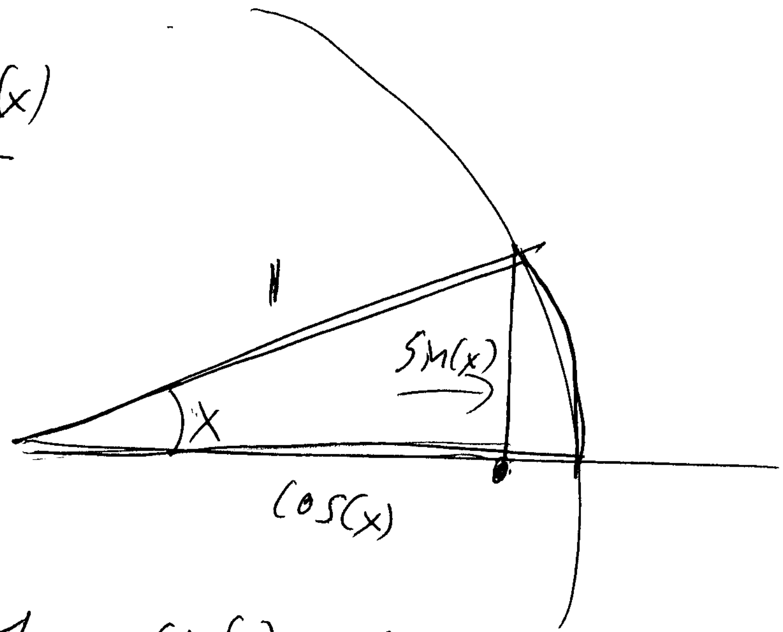


$$\lim_{x \to 0} \quad x^2 \sin\left(\frac{1}{x}\right) = g(x)$$

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1 \qquad f(x) = -x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2 = h(x)$$



$$\lim_{x\to 0^+} \frac{\sin(x)}{x}$$



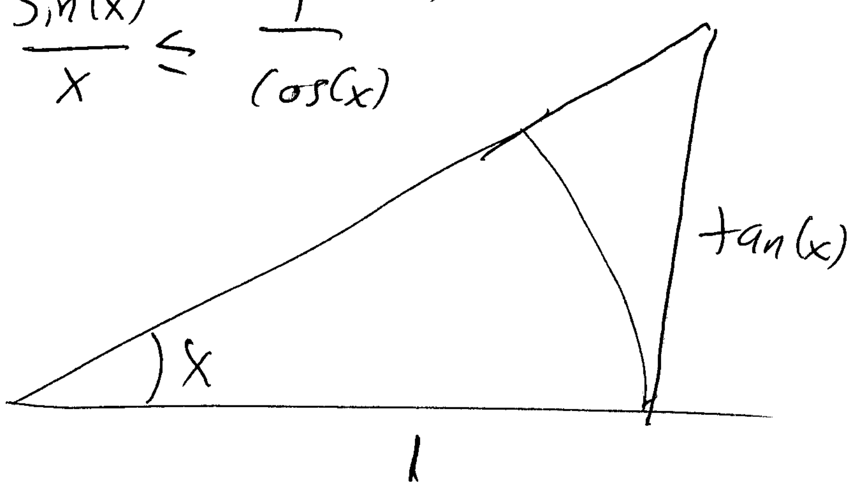
Area of $\triangle$ = $\frac{\sin(x)\cos(x)}{2}$

Area of pie wedge = $\frac{x}{2\pi}\cdot$ Area of circle $= \frac{x}{2}$

$$= \frac{x}{2\pi}\cdot(\pi 1^2) = \frac{x}{2}$$

$$\frac{\sin(x)\cos(x)}{2} \le \frac{x}{2} \qquad \sin(x) \le \frac{x}{\cos(x)}$$

$$\overset{g(x)}{\frac{\sin(x)}{x}} \le \overset{h(x)}{\frac{1}{\cos(x)}}$$



Area $= \frac{1}{2}\,\frac{\sin(x)}{\cos(x)}$

$$\frac{x}{2} \le \frac{1}{2}\frac{\sin(x)}{\cos(x)} \qquad \cos(x) \le \frac{\sin(x)}{x}$$

$$\cos(x) \le \frac{\sin(x)}{x} \le \frac{1}{\cos(x)}$$

$f(x)$ $\quad g(x)$ $\quad h(x)$

Since $\lim_{x\to 0} \cos(x) = \lim_{x\to 0} \frac{1}{\cos(x)} = 1$,

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

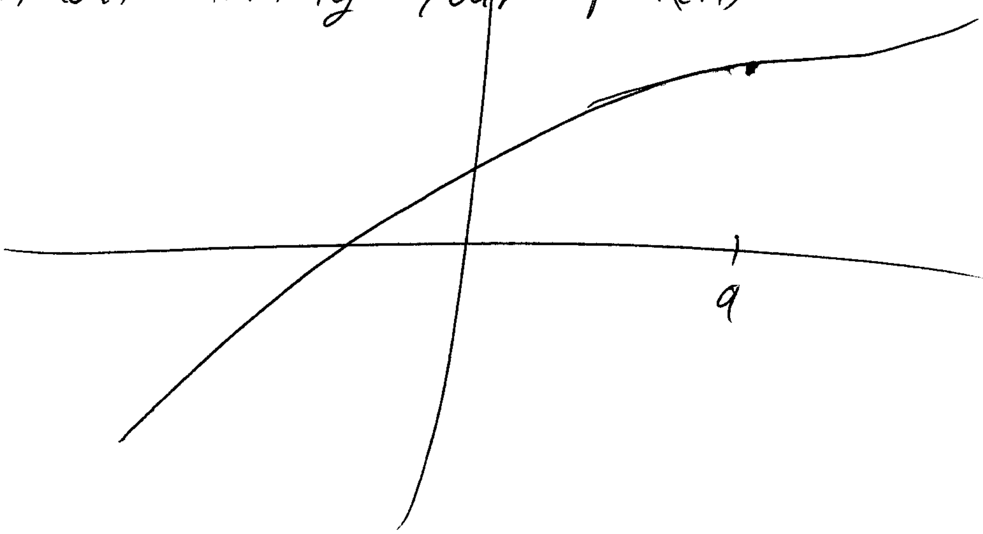
A function $f(x)$ is continuous at a

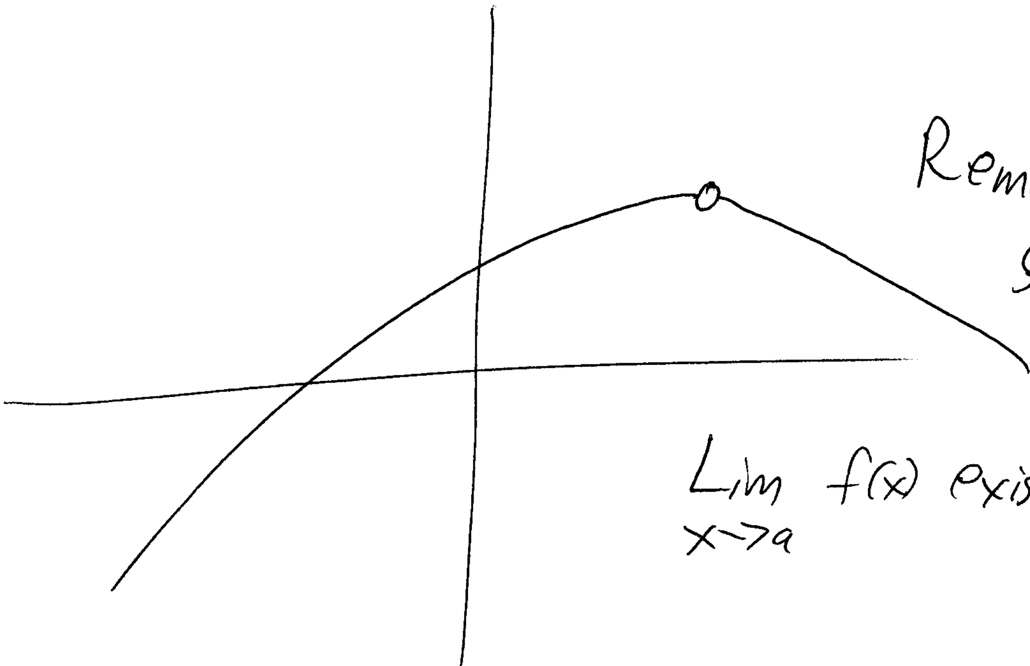
if $\lim_{x \to a} f(x) = f(a)$

Ex: Polynomials, Ratios of polys w/ denominator $\neq 0$.

e^x, $\ln(x)$ if $x>0$.

You can graph a continuous function without lifting your pencil)

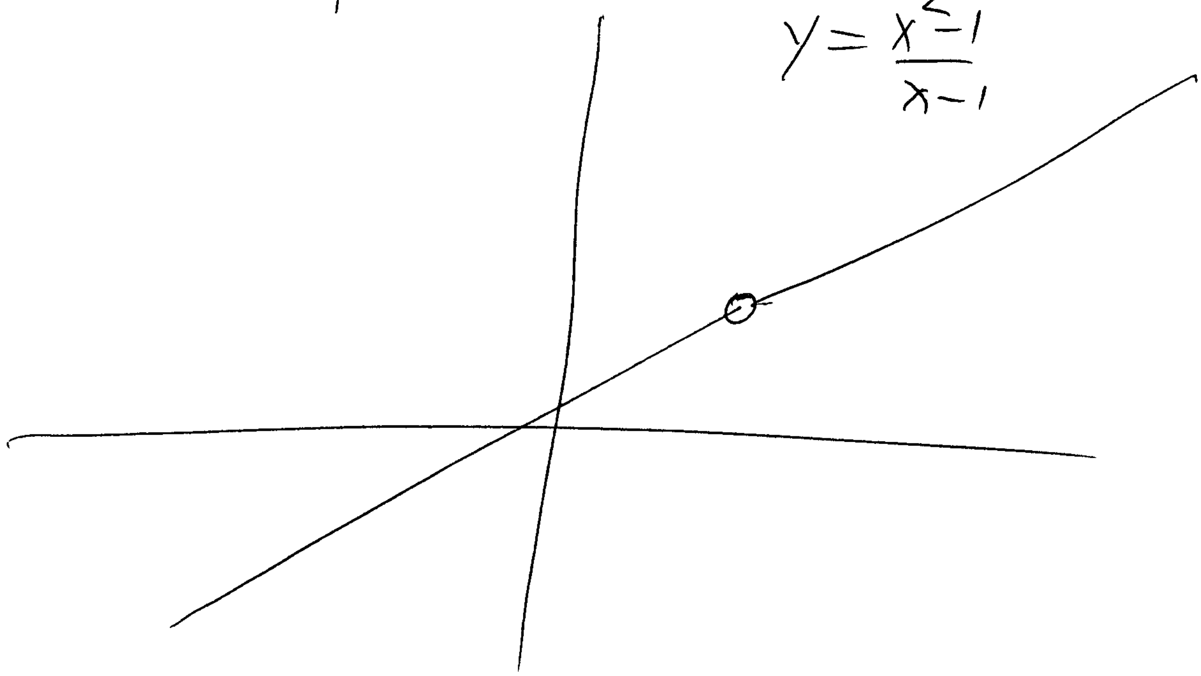




Removable ~~Singul~~ discontinuity

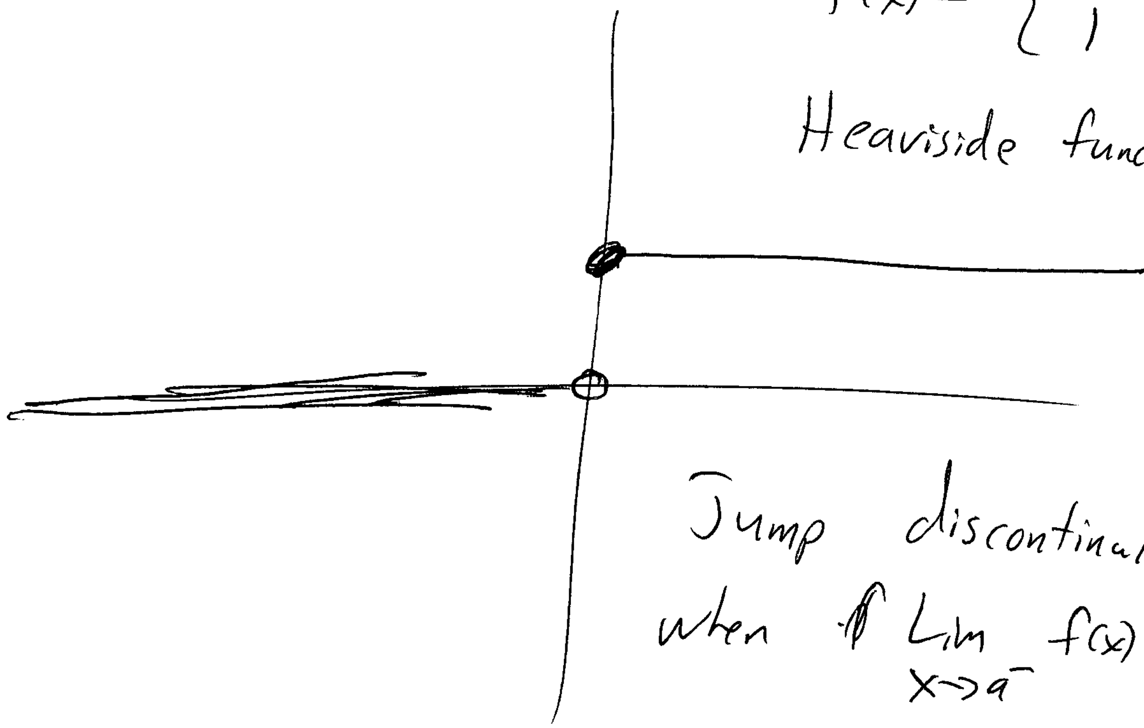
$\lim_{x \to a} f(x)$ exists, but $\neq f(a)$

$y = \frac{x^2 - 1}{x - 1}$

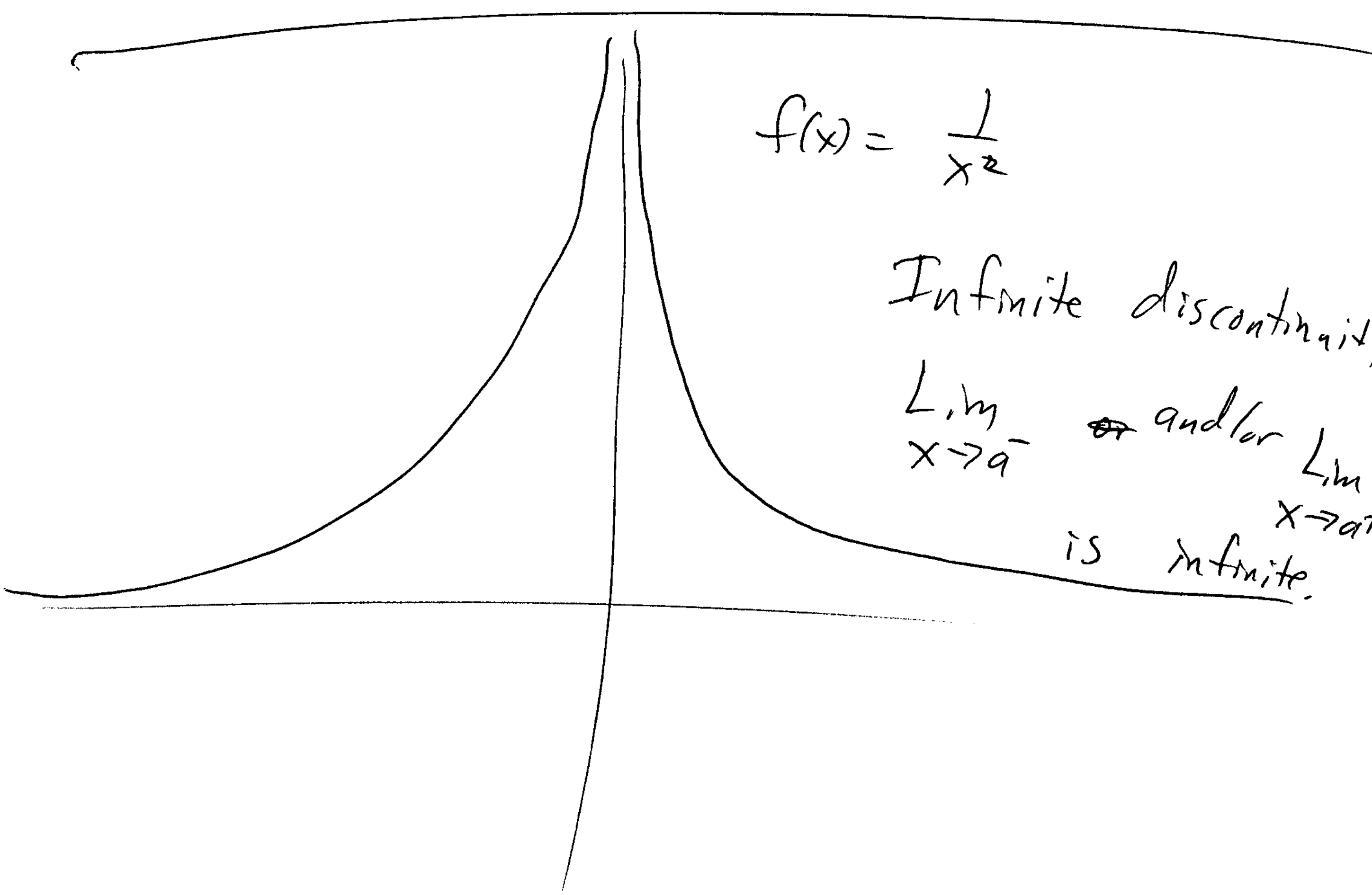


$$f(x) = \begin{cases} 0 & x < 1 \\ 1 & x \geq 0 \end{cases}$$

Heaviside function.

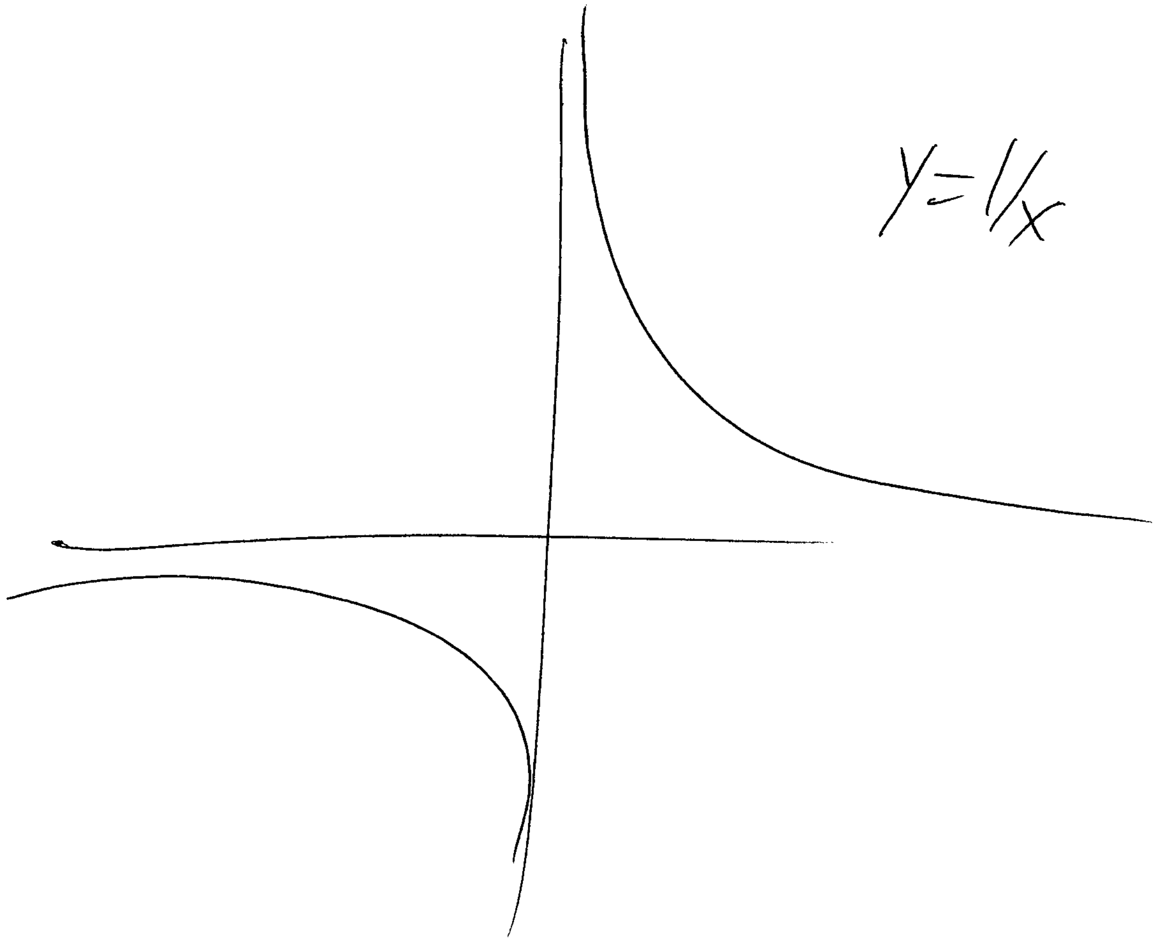


Jump discontinuity,
when $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ disagree.



$$f(x) = \frac{1}{x^2}$$

Infinite discontinuity
$\lim_{x \to a^-}$ ~~or~~ and/or $\lim_{x \to a^+}$ is infinite.


y=1/x