M408N Worksheet, Due Monday, September 17

1) For the following, find the limits $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a} f(x)$, if they exist.

a) $a = 4$ and $f(x) = \frac{|x-4|}{x-4}$.

b) $a = 2$ and $f(x) = \frac{|x-2|}{x}$.

2) Sketch the graph of a continuous function f with all of the following properties:

a) $f(0) = 2$

b) $f(x)$ is decreasing for $0 < x < 3$

c) $f(x)$ is increasing for $3 < x \leq 5$

d) $f(x)$ is decreasing for $x > 5$.

e) $f(x) \to 9$ as $x \to \infty$.

3) How can you remove the discontinuity of f at $x = 2$, where

$$f(x) = \frac{x^3 - 4x^2 - 11x + 30}{x^2 - 4}.$$

In other words, find a function $g(x)$ that is continuous at $x = 2$ and that equals $f(x)$ everywhere except at $x = 2$.

4) For the following two functions, show that there is a number c between 0 and 1 such that $f(c) = 0$.

a) $f(x) = e^x - 3x$

b) $f(x) = 2^x - \frac{1}{x}$.

5) Let $f(x) = x^3 - 3x^2 - 4x + 12$, and let $h(x) = \begin{cases} \frac{f(x)}{x-3} & x \neq 3 \\ p & x = 3 \end{cases}$, where p is an appropriately chosen constant.

a) Find all the zeroes of f.

b) Find the value of p that makes g continuous at $x = 3$. Explain.