M408R Makeup First Exam, Due October 3, 2014 at the beginning of class

This is an open-book, open-notes, all-calculators-allowed take-home exam. You are welcome to look at the solutions to the original first midterm. In fact, thinking about the first midterm, and the problems in it, is the best way to do well on this exam. You are also welcome to talk with Dr. Sadun, with Clark, or with any of the staff at Calc Lab about the first midterm and its solutions.

However, you are **not** allowed to talk to anybody about the problems on this makeup exam. You solutions must be 100% your own work, with **no** input from anybody else.

If your score on this makeup exam is better than your score on the original exam, I will replace the original exam score with the average of the two. Hopefully that will get you half-way to 100%.

Please sign the following honor statement on the dotted line, and then print your name clearly below the line.

All work presented here is my own. I have neither given nor received any help with these problems, and I will report any violation of this pledge by others.

Problem	Score
1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

1) Let $P(t)$ be the number of patients in a certain hospital on day t, and suppose that the rate equation for P is

$$P' = 20 - \frac{1}{8}P$$

and that there are 200 patients on October 7.

a) Use Euler's method with step size $h = 2$ to estimate the number of patients on October 9.

b) Use Euler's method with step size $h = 2$ to estimate the number of patients on October 5.

c) Use Euler's method with step size $h = 1$ to estimate the number of patients on October 9.

2) Here is a table of values of a function $f(x)$.

x	$f(x)$
9.96	0.99825933842
9.97	0.99869515831
9.98	0.99913054128
9.99	0.99956548822
10	1.00000000000
10.01	1.00043407748
10.02	1.00086772153
10.03	1.00130093302
10.04	1.00173371281

a) Find $f'(10)$ to within 0.0001.

b) Use this information and the microscope equation to estimate $f(10.25)$.

3) Suppose that $f(x)$ is a differentiable function with $f(5) = 18$ and $f'(5) = 3$.
a) Find an equation for the line tangent to $y = f(x)$ at $(5, 18)$.

b) Estimate the values of $f(5.03)$ and $f(4.92)$.

4) Ozone is an unstable form of oxygen, with molecular formula O_3. Let $C(t)$ be the concentration of ozone in the upper atmosphere at time t. Ozone is created when cosmic rays hit the upper atmosphere. Ozone is destroyed whenever two ozone molecules collide. [Note: in reality there are other ways to destroy ozone, but we'll ignore those in this problem.]

a) How does the rate of ozone creation depend on C? (E.g. is it independent of C? Proportional to C? Proportional to C^2? Some other expression?)

b) How does the rate of ozone destruction depend on C (Constant? Proportional to C? Proportional to C^2? Other?)

c) Write down a rate equation for C. This rate equation should involve two unknown parameters, one having to do with ozone creation and one with ozone destruction.

5) a) Find the derivative of the function $f(t) = 3t^2 - 12t + 14$. You may use the formulas from Section 3.5.

b) The position of a particle is given by $x(t) = 3t^2 - 12t + 14$, where t is measured in seconds and x is measured in feet. How fast is the particle moving at time $t = 1$? Is it moving forwards or backwards?

c) At what time(s) is the particle's velocity equal to zero?