M408R Second Midterm Exam, October 24, 2014

1) This problem is a series of short questions, each worth 6 points (a–e) or 5 points (f and g).

a) Compute df/dx, where $f(x) = 1027e^{2x}$.

b) Compute dh/dx, where $h(x) = \ln(\cos(2x))$

c) Compute the derivative of $j(x) = \sin(x)e^{(x^2)}$.

d) Compute the derivative of $k(x) = \frac{2x+1}{3x+7}$.

e) Find the partial derivative of $F(x, y) = \frac{e^{xy}}{y}$ with respect to x.

f) Simplify $\sqrt{\ln(e^{25})}$ as much as possible.

g) Simplify $4^{\log_2(7)}$ as much as possible.

2) Consider the function $f(x) = \ln(x^2 + 1) - \ln(5)$.

a) Compute $f'(x)$.

b) Let L be the line tangent to the curve $y = f(x)$ at $x = 2$. Find the slope of L.

c) Find the equation of L. (You can leave your answer in either point-slope or slope-intercept form.)

d) Use your answers to (b) and (c) to approximate $f(2.05)$. [Note: This is a problem about calculus, so just plugging $x = 2.05$ into your calculator and hitting the ln key is NOT worth any points. But it's not a bad way to check your answer.]

3) A colony of bacteria is undergoing exponential growth. That is, if $B(t)$ is the amount of bacteria (measured in grams) at time t (measured in hours), then

$$B(t) = Ce^{rt},$$

where C and r are unknown constants.

a) If there are 2 grams of bacteria at time $t = 0$, what is C?

b) 4 hours later, the mass of bacteria has grown to 2.83 grams. From this fact (and your answer to (a)), compute r.

c) At some time later, there are 4 grams of bacteria. How fast (in grams/hour) is B changing at this later time?

4) (Note: the following numbers are made up, but the problem of animal extinction is very real.) The population $E(t)$ of elephants in East Africa is decreasing at a rate proportional to the existing population. In the year 2000, there were 35,000 left. At that time, their number was decreasing at a rate of 1400 elephants/year.

a) Write down an initial value problem that governs the population of elephants. That is, write down i) a rate equation and ii) an initial condition. Your answers should be of the form
$E'(t) =$(some expression involving $E(t)$), E(some time) = (some value).

b) Write down the *solution* to this initial value problem. Your answer should be of the form $E(t) =$ (some explicit function of t).

c) If this model is correct, how many elephants will be left in 2050? In 2100?