

M408R Third Midterm Exam, November 21, 2014

1) Compute the following integrals, each worth 6 points. Note that they don't all use the same technique! Please indicate what technique you are using.

a) $\int \cos(z)(1 - \sin^2(z))dz.$

b) $\int \frac{dw}{3w + 1}.$

c) $\int_0^1 \frac{2.4}{t^2 + 1} dt.$

d) $\int x^2 + \frac{1}{x^2} dx.$

e) $\int_1^e \frac{\ln(r)}{r} dr.$

f) $\int (y^2 + 1)^2 dy.$

g) $\int_{-1}^3 |x| dx.$ [Hint: Break it up as the sum of two definite integrals.]

2) Using anti-derivatives. Let $a(t) = -9.8$.

a) Find an anti-derivative $v(t)$ of $a(t)$ such that $v(1) = 39.2$. (There is one and only one correct answer.)

b) Find an anti-derivative $z(t)$ of $v(t)$ such that $z(1) = 44.1$.

c) For what values of t is $z(t) = 0$?

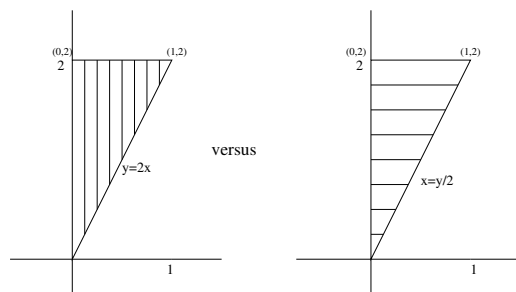
d) A cannonball is shot into the air. At time $t = 1$ second, it is 44.1 meters above the ground and is rising at 39.2 meters/second. At all times, it is accelerating *downwards* at $9.8m/s^2$ because of gravity. At what time will it hit the ground? At what time was it fired?

3) A snowstorm is expected to hit the town of Frostbite Falls tonight, starting at midnight. The rate at which snow falls is expected to be given by the function $r(t) = 5e^{-2t}$ inches/hour, where t is the number of hours since midnight. (E.g., 1:30 AM is $t = 1.5$, so at 1:30 AM the snow is coming down at a rate of $5e^{-3}$ inches/hour)

a) Write down a definite integral that gives the predicted total snowfall between midnight and 5AM. **Explain your reasoning!**

b) Now compute that integral. Give your answer first as an exact expression involving e , and then plug that expression into your calculator to get a numerical answer.

4) (2 pages!) Let T be a triangle in the x - y plane with vertices at the origin, at $(0, 2)$, and at $(1, 2)$. In other words, it is bounded by pieces of the y -axis, the line $y = 2$, and the line $y = 2x$ (aka $x = y/2$). We are going to compute the area of T in two different ways.



a) Write down an integral over x that gives the area of T . Explain where this integral comes from!

Your answer in (a) should encode what happens when you cut T up into a bunch of vertical strips, add up the approximate area of each strip, and take a limit (as in the first figure). Instead, imagine cutting T up into N **horizontal** strips, all of the same height Δy , as in the second figure.

b) What is the width of each strip, as a function of y ?

c) What is the approximate area of each strip?

d) Write down a Riemann sum (based on this approach) that gives the approximate area of T .

e) Now write down an integral that is the limit of this Riemann sum as we slice things finer and finer. [Voila! You've found a second way to compute areas by integration.]