Individual Homework #6: Due October 29, 2014

Please **read** Section 4.4.

A. Please **do** exercises exercises 3, 4, 5, 6, 10, 11, 15, and 16, in Section 4.4 (pp. 257–260).

Some hints and notes on these exercises:

- In general, where population growth is mentioned, assume unlimited (exponential) growth unless otherwise specified.
- Hint for exercises 15 and 16, pp. 259–260: GUESS that your solution is of the form $C\ln(x) + D$ (or $C\ln(t) + D$, in the case of exercise 16(b)), for constants C and D. Then figure out which constants C and D will work, using information in the given exercise.

B. The arctangent and Ebola.

Recall that, in class on 2/28, we studied the arctangent function $\arctan(x)$, and in particular, we saw that

$$\frac{d}{dx}\Big[\arctan(x)\Big] = \frac{1}{1+x^2}.$$

We are now going to consider an application of the arctangent function to the study of an actual 90-day-long outbreak of Ebola in the Democratic Republic of Congo (DRC), in 1995.

The t-th entry in the list below gives the *actual number* $R(t)$ of total deaths from this Ebola outbreak, at the end of day t (starting with $t = 1$ and continuing to $t = 90$). (The "R" in $R(t)$ stands for "reposed," which means "dead." It also stands for "real," since this is the real data.)

1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 3, 4, 6, 9, 11, 13, 13, 13, 13, 13, 13, 15, 16, 17, 19, 23, 24, 26, 28, 33, 33, 33, 33, 34, 36, 38, 43, 45, 52, 61, 74, 88, 99, 104, 112, 119, 120, 121, 124, 130, 135, 136, 146, 153, 159, 165, 170, 176, 179, 182, 183, 185, 187, 191, 192, 193, 193, 194, 195, 196, 200, 204, 207, 208, 208, 210, 210, 210, 211, 211, 211, 211, 211, 214, 214, 214, 214, 214, 214, 214

We are now going to study $R(t)$ and its relation to the arctangent function, using MATLAB.

1. Cumulative deaths.

i. Open up the MATLAB file Ebola.m, available on our course page, and run it (just enter Ebola in Command Window). You should get a plot consisting of a bunch of dots. These dots represent the above numbers $R(t)$ on the vertical axis, plotted against t on the horizontal axis.

You DON'T have to supply a printout of this graph, but DO answer this question: what is the basic shape of the graph suggestive of? That is: what important type of function does it look like? Hint: we just mentioned such a function above.

ii. Now plot the function

$$M = \frac{1654}{21}\left(\arctan\left(\frac{2(t-45)}{21}\right) + \arctan\left(\frac{30}{7}\right)\right)$$

in MATLAB, by entering the following:

```
M = ezplot((1654/21)*(atan(2*(t-45)/21)+atan(30/7)),[1,90])
```

Make sure you type it in exactly as above. You don't need to supply a printout, but make sure it works. (You should get a nice, smooth curve that looks a lot like the arctangent curve we saw in class.) Do not close this plot as we will place the data plot on this figure.

iii. We now wish to compare your graphs in parts (i) and (ii) above, by putting these graphs onto the same set of axes. This is easy: type

```
hold on, Ebola
```

into the MATLAB Command Window. Once you've done this, DO supply a copy of your graph with this assignment.
What are the major differences, if any, between the two graphs? Do you think the arctangent function can be useful in studying the spread of disease?

2. **The Ebola death rate.** If the function $M(t)$ above models cumulative deaths in the above Ebola outbreak, then the *death rate*, in individuals/day, can be modeled by the *derivative* dM/dt.

i. Use differentiation rules (including the chain rule), along with the formula for $d\big[\arctan(x)\big]/dx$ that you found above, to show that

$$\frac{dM}{dt} = \frac{3308}{21^2 + 4(t-45)^2}$$

(you'll need some algebra to get your answer into this form). Please show all of your work.

ii. Type the following:

```
ezplot('3308/(21^2 +4*(x-45)^2)',[1,90]),xlabel('t days'),ylabel('dM/dt
(individuals per day)')
```

(all on one line—we just couldn't fit it on one line here) into MATLAB, execute this line, and provide a copy of your printout with this HW.

iii. Fill in the blanks – in each blank *except for the last one*, the correct answer is *either* the letter "S," or the word "bell" (note: it's not necessary to answer in complete sentences here; you can just supply a list of the appropriate responses, in the right order. OR, you can print out the paragraph below and supply it with your completed assignment, with the blanks filled in):

The cumulative death function for the above Ebola outbreak can be modeled fairly well by an arctangent curve, which has something of an elongated ____________ shape. The death rate for the outbreak can then be modeled fairly well by a curve that has something of a/an____________ shape.

OK, so, the derivative of a/an ____________ curve is a/an ____________ curve. Where have we seen this before? We've seen it in SIR!! Remember that, there, the variable R (recovered) followed a/an ____________ curve, while the variable I (infected) followed a/an ____________ curve. But I *is* the derivative of R, or more precisely I is *proportional to* the derivative of R, by the third of the SIR equations, which says $R' =$____________.