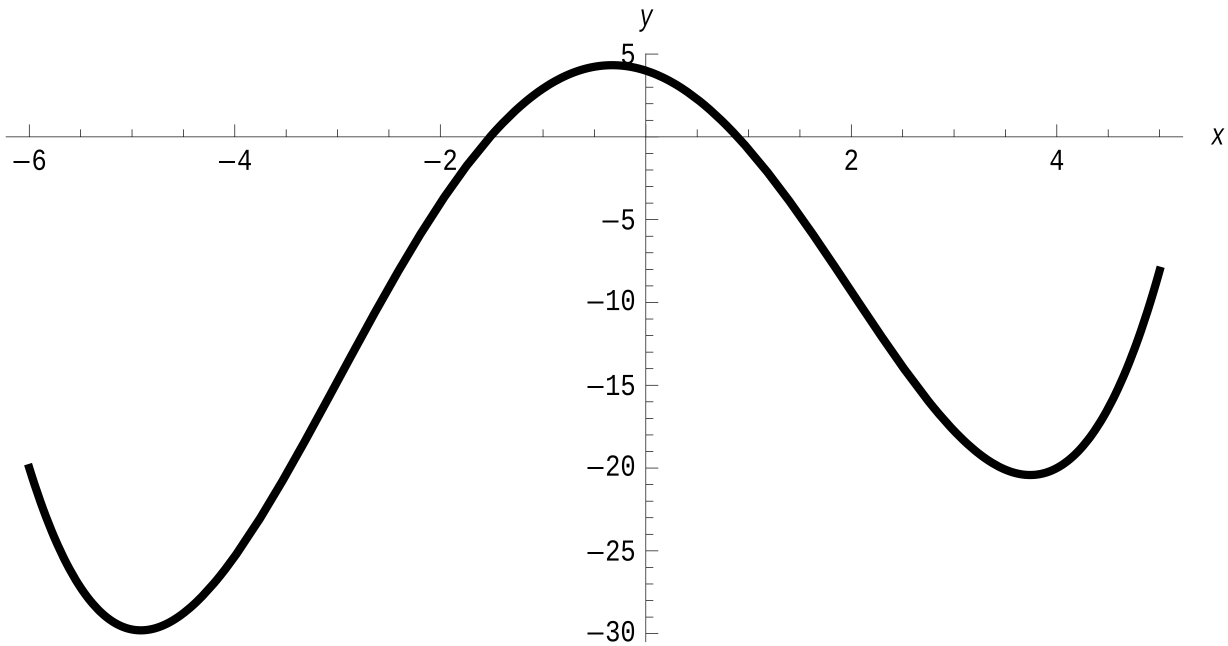
On the axes below is the graph of a certain function $f(x)$.



1. By inspection (just eyeball it), mark the points where the graph of $f(x)$ is *steepest*. Do not include the endpoints of the graph (we're imagining that the graph continues on forever in both directions).

2. Fill in the blanks below: each blank is to be filled in with one of the following terms (some terms may be used more than once, and some not at all):

   derivative   positive   negative $f(x)$ zero

   To say that the graph of $f(x)$ is at its steepest is to say that the slope, or in other words the ________________, of $f(x)$ is bottoming out (as negative as it can get) or peaking (as _______________ as it can get). But we've seen before that, when a function bottoms out or peaks, its ________________ equals zero. SO: to say that the graph of $f(x)$ is at its steepest is to say that the ________________ of the ________________ of $f(x)$ equals ____________.

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3. Find the derivative $f'(x)$ of the function $f(x)$ graphed above, given that

$$f(x) = \frac{x^4}{12} + \frac{x^3}{6} - 3x^2 - 2x + 4.$$

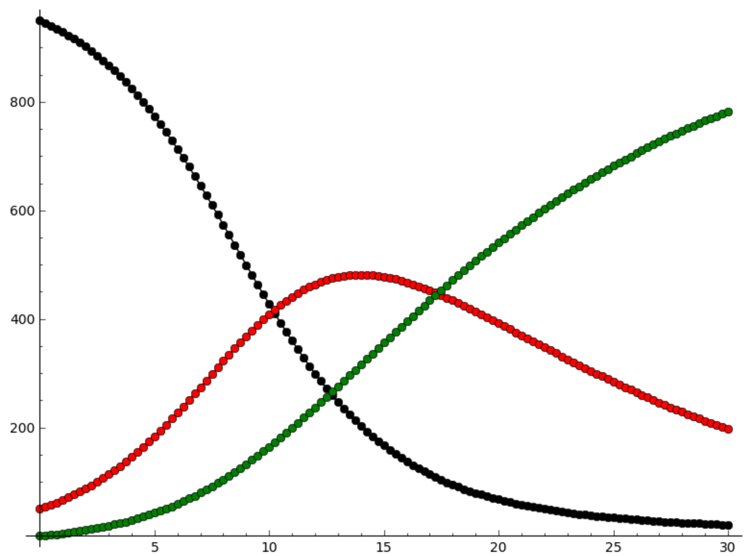
4. For $f(x)$ as above, find $f''(x)$, which is called the *second derivative* of $f(x)$, and means the derivative of the derivative of $f(x)$. (That is, $f''(x) = \frac{d}{dx}[f'(x)]$.)

5. Solve the equation $f''(x) = 0$ for x.

6. Do your results from exercises 1 and 5 above agree? If not, do you want to adjust one of those answers? Please explain.

7. Consider a disease evolving according to the usual SIR equations

$$\begin{aligned} S' &= -a\,S\,I, \\ I' &= \quad a\,S\,I - b\,I, \\ R' &= \qquad\qquad b\,I. \end{aligned}$$



   (a) Explain carefully why (as the graph suggests) R is at its steepest at the very point where I peaks. Hint: use the equation for R' to obtain an equation for R''. Then use ideas from the exercises above.

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(b) By differentiating the above equation for S' (using the product rule), and then plugging in for S' and I' on the right hand side of your result, show that

$$S'' = -a^2\,S^2\,I + ab\,S\,I + a^2\,S\,I^2.$$

(c) Explain why, when S is at its steepest,

$$S - I = \frac{b}{a}.$$