1. Carbon 14 is an isotope of carbon that is formed when radiation from the sun strikes ordinary carbon dioxide in the atmosphere. Thus plants such as trees, which get their carbon dioxide from the atmosphere, contain small amounts of carbon 14. Once a particular part of a plant has been formed, no more new carbon 14 is taken in by that part and the carbon 14 in that part slowly decays. Let P be the percentage of carbon 14 that remains in a part of a tree that grew t years ago.

   (a) The half-life of carbon 14 is about 5750 years. If $P = 100e^{-kt}$, find k.

$$
\begin{aligned}
50 &= 100e^{-k\cdot 5750} \\
\frac{1}{2} &= e^{-5750k} \\
\ln\left(\frac{1}{2}\right) &= \ln\left(e^{-5750k}\right) \\
-\ln(2) &= -5750k \\
k &= \frac{\ln(2)}{5750}
\end{aligned}
$$

   (b) The oldest living trees in the world are the bristlecone pines in the White Mountains of California. Four thousand growth rings have been counted in the trunk of one of these trees, meaning that the innermost ring grew 4000 years ago. What percentage of the original carbon 14 would you expect to find remaining on this innermost ring?

$$
\begin{aligned}
P &= 100e^{-\frac{\ln(2)}{5750}\cdot 4000} \\
&= 100e^{-0.482189} \\
&= 100 \cdot 0.61743 \\
&= 61.74 \text{ percent.}
\end{aligned}
$$

(c) A piece of wood claimed to have come from Noah's Ark is found to have 48.37 % of the carbon 14 remaining. It has been suggested that the Great Flood occurred in 4004 B.C. Is the wood old enough to have come from Noah's Ark?

Let's find out how old the piece of wood is.

$$48.37 = 100e^{-\frac{\ln(2)}{5750}t}$$

$$0.4837 = e^{-\frac{\ln(2)}{5750}t}$$

$$\ln(0.4837) = -\frac{\ln(2)}{5750}t$$

$$t = \ln(0.4837) \cdot -\frac{5750}{\ln(2)}$$

$$t \approx 6025 \text{ years.}$$

$2013 - 6025 = 4012$ B.C., so yes, it could have come from the ark.

2. Bacteria in a lab culture grow in such a way that the instantaneous rate of change of bacteria is directly proportional to the number of bacteria present. Let B represent the number of bacteria present (in millions) at time t hours.

(a) Write an initial value problem that expresses the relationship between B' and B.

$B' = kB$.

(b) Suppose that initially there are 5 million bacteria. Three hours later, the number of bacteria has grown to 7 million. Find a formula for B as a function of t.

We'll start with the unlimited growth solution $B = B_o e^{kt}$. We know the initial quantity is 5 million, so $B_o = 5$. Now, to find k, we'll use that $B(3) = 7$:

$$7 = 5e^{k \cdot 3}$$

$$\frac{7}{5} = e^{3k}$$

$$\ln\left(\frac{7}{5}\right) = 3k$$

$$k = \frac{\ln\left(\frac{7}{5}\right)}{3} \approx 0.1121574.$$

(c) What will the bacteria population be one full day after the first measurement?

$$B(24) = 5e^{0.1121574 \cdot 24} \approx 73.8 \text{ million.}$$

(d) What will the rate of growth B' of the population be, one full day after the first measurement? Remember $B' = kB$, so

$$B' = 0.1121574 \cdot 73.8 \approx 8.3 \text{ million bacteria per hour,}$$

where the 73.8 is the population at $t = 24$ that we found in part (c).

(e) When will the population reach 1 billion (1000 million)?

$$\begin{aligned} 1000 &= 5e^{0.1121574t} \\ 200 &= e^{0.1121574t} \\ \ln(200) &= 0.1121574t \\ t &= \frac{\ln(200)}{0.1121574} \\ t &\approx 47.2 \text{ hours.} \end{aligned}$$

3. According to *Moore's Law* (formulated by Gordon Moore in 1965), the maximum number N of transistors that can be fit on a microchip increases exponentially with time. It is known that, in 1971, $N = 2{,}250$, and in 2000, $N = 42{,}000{,}000$.

   (a) Assuming that Moore's Law is correct, find a formula for N as a function of time t, in years.

   We have

$$N = N_0 e^{kt} = 2250\, e^{kt}.$$

   To find k, we use the fact that $N(29) = 42,000,000$, so that

$$\begin{aligned} 42,000,000 &= 2250 e^{k\cdot 29} \\ \frac{42,000,000}{2250} &= e^{k\cdot 29} \\ \ln\left(\frac{42,000,000}{2250}\right) &= k \cdot 29 \\ k &= \ln\left(\frac{42,000,000}{2250}\right) \Big/ 29 = 0.339121... \end{aligned}$$

   So the formula is

$$N = 2250\, e^{0.339121t}.$$

   (b) Using Moore's Law, predict N for the year 2013. How does this compare with the actual N, which is about 5 billion, for that year?

$$N(42) = 2250\, e^{0.339121\cdot 42} \approx 3.450370 \;\; \text{billion.}$$

   This is at least in the same ballpark as the true number of 5 billion.