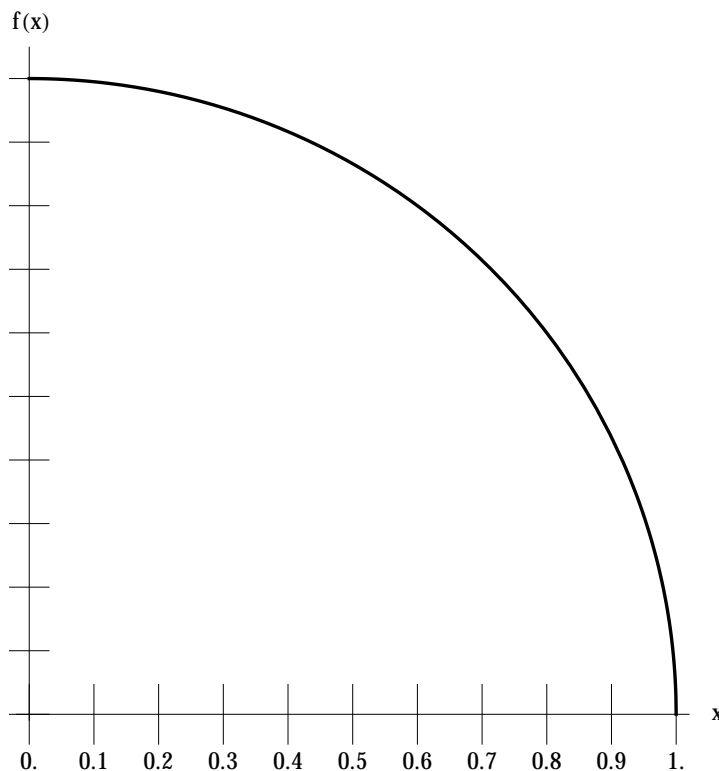


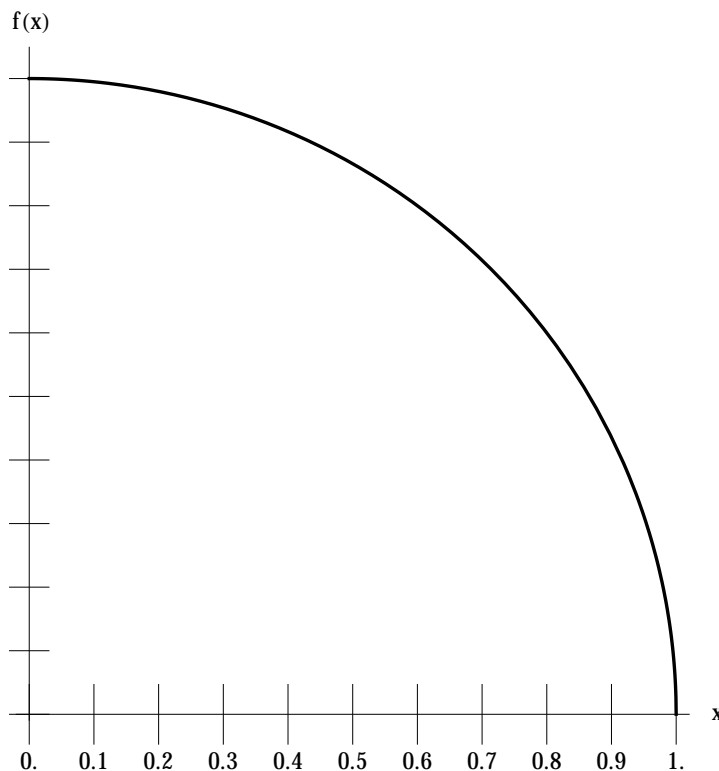
1. Below is a sketch of the function $f(x) = \sqrt{1 - x^2}$.



- (a) On top of this sketch, draw in the rectangles that would represent a *left endpoint* Riemann sum approximation, with $n = 5$, to the area A under this graph, from $x = 0$ to $x = 1$.
- (b) Will your above left endpoint Riemann sum approximation, call it $\text{LEFT}(5)$, be an overestimate or an underestimate of the above area? Explain, without doing any computations (yet).
- (c) Use the Riemann sum represented in your above picture to approximate A . Give a numerical answer with at least four digits to the right of the decimal point.

$$A \approx \text{LEFT}(5) =$$

2. Below, again, is a sketch of the function $f(x) = \sqrt{1 - x^2}$.

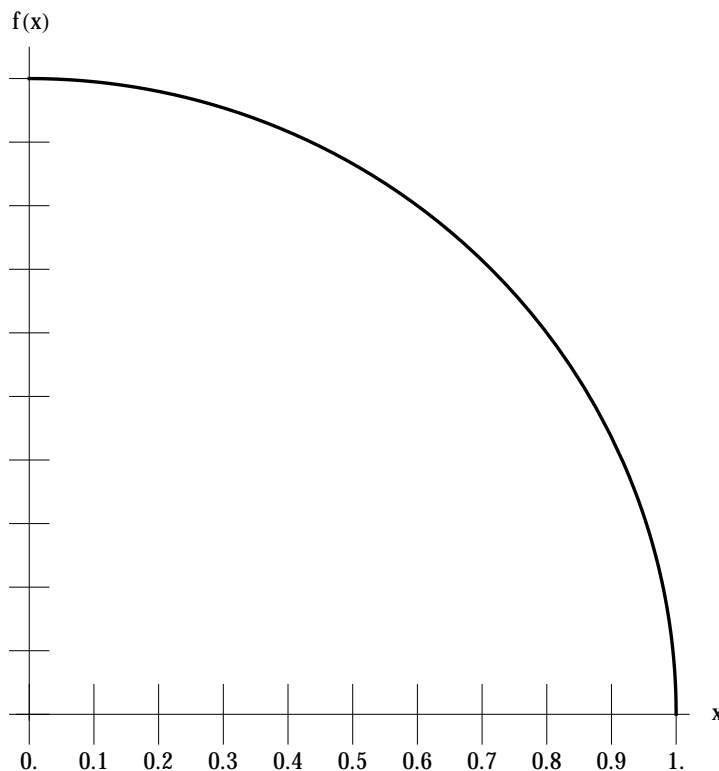


- (a) On top of this sketch, draw in the rectangles that would represent a *right endpoint* Riemann sum approximation, with $n = 5$, to the area A under this graph, from $x = 0$ to $x = 1$.
- (b) Will your above left endpoint Riemann sum approximation, call it $\text{RIGHT}(5)$, be an overestimate or an underestimate of the above area? Explain, without doing any computations (yet).

- (c) Use the Riemann sum represented in your above picture to approximate A . Give a numerical answer with at least four digits to the right of the decimal point.

$$A \approx \text{RIGHT}(5) =$$

3. Below, yet *again*, is a sketch of the function $f(x) = \sqrt{1-x^2}$.



- (a) On top of this sketch, draw in the rectangles that would represent a *midpoint* Riemann sum approximation, with $n = 5$, to the area A under this graph, from $x = 0$ to $x = 1$.
- (b) Will your above left endpoint Riemann sum approximation, call it MID(5), be an overestimate or an underestimate of the above area? Explain, without doing any computations (yet).
- (c) Use the Riemann sum represented in your above picture to approximate A . Give a numerical answer with at least four digits to the right of the decimal point.

$$A \approx \text{MID}(5) =$$

4. *Without actually computing* the quantities LEFT(2000), RIGHT(2000), and MID(2000), list the following in increasing order:

LEFT(2000), RIGHT(2000), MID(2000), MID(5), A , RIGHT(5), LEFT(5)

Please explain how you got your answer.

5. What is A *exactly* (to four decimal places)? Hint: as you can see, the graph of $f(x)$ on $[0, 1]$ describes a quarter circle.