In this worksheet, we practice integration by evaluating the definite integral

$$\int_0^{\pi/2} \cos^2(x)\,dx$$

in three different ways.

We will need the following two important trig identities:

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x)); \qquad \cos^2(x) + \sin^2(x) = 1.$$

1. Evaluate

$$\int_0^{\pi/2} \cos^2(x)\,dx$$

by rewriting the integrand, using the first of the above trig identities, and then doing the integration directly (using the Fundamental Theorem of Calculus). Express your answer in terms of π.

2. (a) Show that, if

$$F(x) = \frac{x + \sin(x)\cos(x)}{2},$$

then $F'(x) = \cos^2(x)$. (You will need the second of the two trig identities stated above to do some simplification after you differentiate.)

(b) Use the result of part (a) above, and the Fundamental Theorem of Calculus, to again evaluate

$$\int_0^{\pi/2} \cos^2(x)\,dx.$$

Again, express your answer in terms of π.

3. (a) Fill in the five blanks: Since $\cos^2(x)+\sin^2(x) = 1$, we have $\cos^2(x) = 1-$____________, and therefore,

$$\cos(x) = \pm\sqrt{1 - \underline{\hspace{3cm}}}.$$

But, since $\cos(x) \geq 0$ on the interval $[0, \pi/2]$, we need to take the ____________ sign here and not the ____________ sign, so we conclude that, on this interval,

$$(*) \qquad \cos(x) = \sqrt{1 - \underline{\hspace{3cm}}}.$$

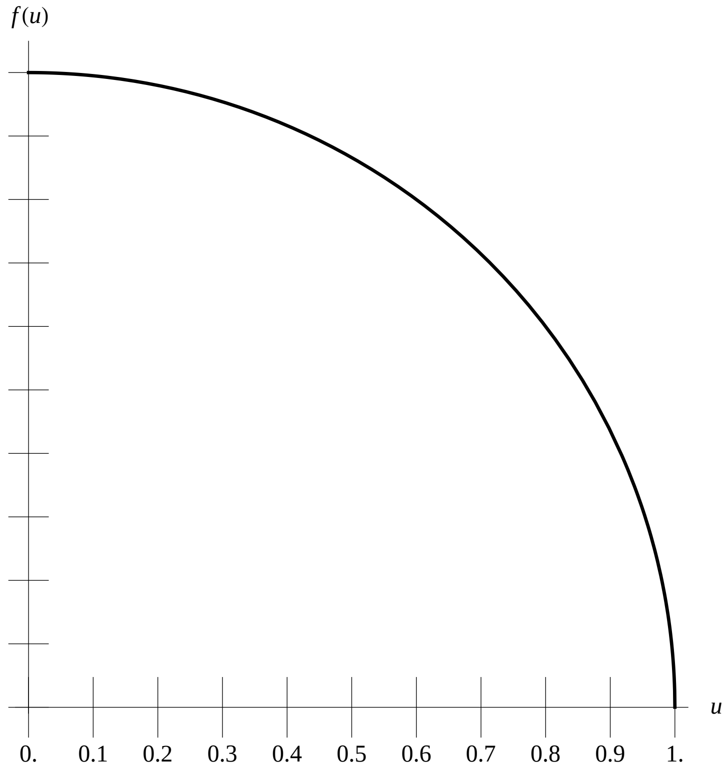
(b) Fill in the blank: using equation $(*)$ above, we find that

$$\int_0^{\pi/2} \cos^2(x)\, dx = \int_0^{\pi/2} \cos(x) \cdot \cos(x)\, dx = \int_0^{\pi/2} \underline{\hspace{3cm}} \cdot \cos(x)\, dx.$$

(c) Substitute $u = \sin(x)$ into the result of part (b) above. Then $du =$____________ dx; also, when $x = 0$, $u =$ ____________, and when $x = \pi/2$, $u =$ ____________. So we find that

$$\int_0^{\pi/2} \cos^2(x)\, dx = \int_0^1 \underline{\hspace{3cm}}\, du.$$

(d) Below is a sketch of the function $f(u) = \sqrt{1-u^2}$.



What is $\int_0^1 f(u)\,du$ *exactly*? (Your answer should involve the quantity π.) Hint: as you can see, the graph of $f(u)$ on $[0,1]$ describes a quarter circle.

(e) Use your answers to parts (c) and (d) above to again evaluate

$$\int_0^{\pi/2} \cos^2(x)\,dx.$$