November 18, 2014 Worksheet 19: Integration by Substitution (indefinite, definite, initial value)

1. Find

$$\int \left(x + \frac{1}{x}\right) \left(1 - \frac{1}{x^2}\right) \, dx$$

in two ways:

(a) by a substitution;

(b) by multiplying out and integrating term by term. Show that your two answers are the same.

2. Find
$$\int \frac{x}{(x^2+1)^2} dx$$

3. Find $\int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$. Express your answer in terms of whole numbers and e.

4. Find the area under the graph of

$$y = \frac{\ln(x)}{x}$$

from x = 1 to $x = e^2$. Express your answer as a whole number.



5. Find
$$\int \frac{1}{5x-1} dx$$

6. Solve the initial value problem

$$\frac{dy}{dx} = \frac{e^x}{5+e^x}, \qquad y(\ln 5) = 4 + \ln 10.$$