

M408R Worksheet #20: Integration by Parts

Back in Worksheet 9, we derived the Product Rule for derivatives:

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$$

Now we're going to turn that inside-out.

1) Using the product rule, find the derivative (with respect to x) of $x \sin(x)$.

2) Now find two functions such that integral of one, plus the integral of the other, adds up to $x \sin(x) + C$.

3) The integral of one of those functions should be easy. Based on your answer to (2), find the integral of the other.

Now let's do the same sort of thing with a more general pair of functions:

4) Find the derivative of $f(x)g(x)$.

5) Find two functions such that the integral of the first, plus the integral of the second, equals $f(x)g(x) + C$.

6) Express the integral of one of these functions in terms of $f(x)$, $g(x)$ and the integral of the other.

If all went right, you should have gotten something like:

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx.$$

(You may have also had a “+C” in there somewhere, but it’s not actually needed, since each of those integrals already involves an arbitrary constant.) Sometimes, $\int f'(x)g(x)dx$ is a lot easier than $\int f(x)g'(x)dx$.

7) By taking $f(x) = \ln(x)$ and $g(x) = x^3$, compute $\int 3x^2 \ln(x)dx$.

8) Sometimes we write “ u ” as shorthand for $f(x)$ and v as shorthand for $g(x)$. Explain what du and dv mean.

9) In terms of u and v , the formula for integration by parts becomes $\int u dv = uv - \int v du$. By taking $u = \ln(x^2 + 1)$ and $dv = 2x$, compute $\int 2x \ln(x^2 + 1) dx$. (Note that v does NOT have to be x^2 . There are other functions whose derivatives are $2x$, and one of them works a lot better.)

10) Sometimes integrating by parts once isn't enough. Use integration by parts with $u = x^2$ to turn $\int x^2 \cos(x) dx$ into an easier integral. Then use integration by parts again (this time with $u = 2x$) to do that second integral.

11) Check your answer to (10) by taking its derivative and comparing the result to $x^2 \cos(x)$. (You may want to do the same with your answers to (7) and (9))