Worksheet 4: SIR continued **September 11, 2014**

Goal: To explore more ideas about modeling with rate equations and SIR.

1. A town of population 100,000 is hit with a measles epidemic, which evolves according to the usual SIR equations

$$\begin{aligned} S' &= -a\,S\,I, \\ I' &= \quad a\,S\,I - b\,I, \\ R' &= \qquad\qquad b\,I. \end{aligned}$$

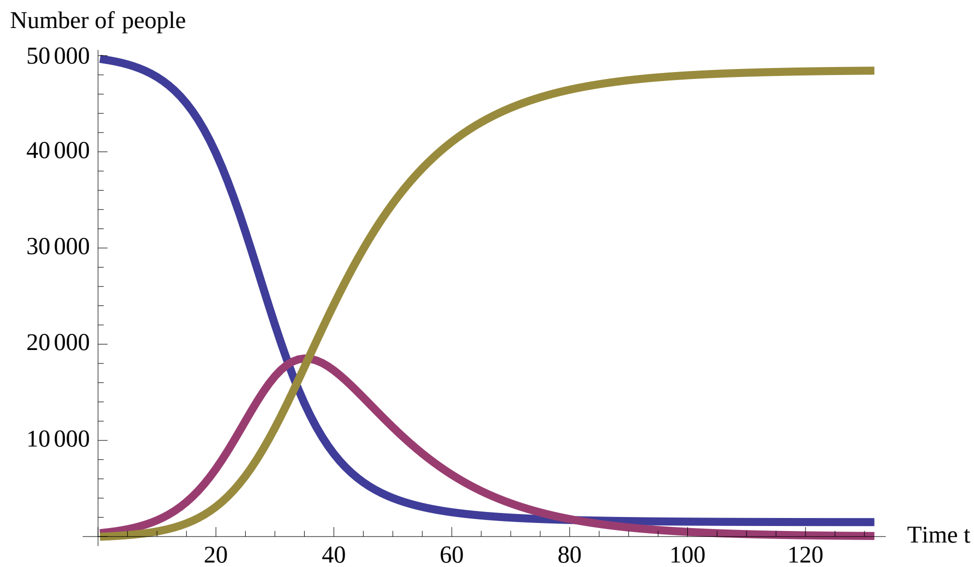
This unique strain of the measles is known to last for twelve days.

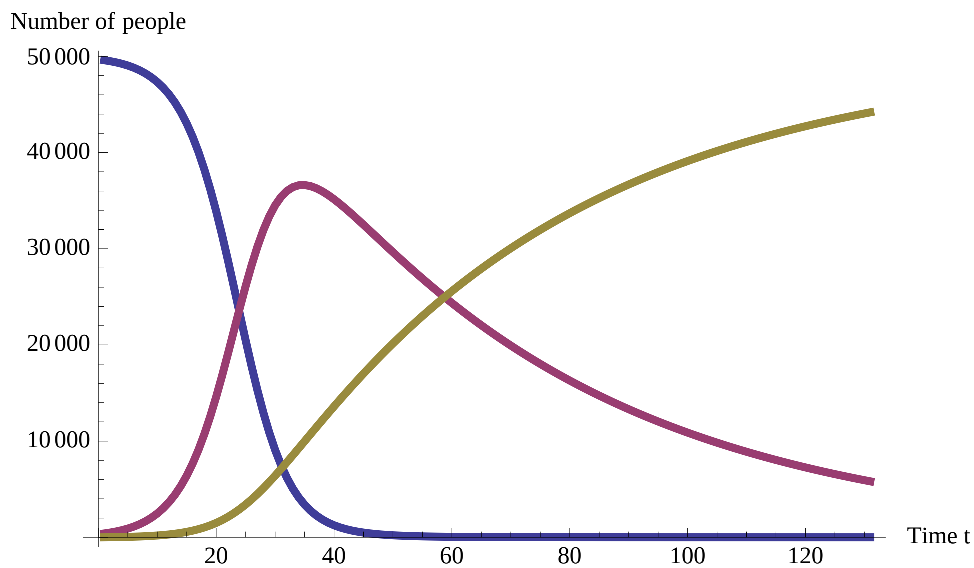
On day 15, 14,893 people are susceptible (that is, $S(15) = 14,893$) and 69,613 people are infected (so $I(15) = 69,613$). *One tenth* of a day later, $S = 13,856$.

Make a possible model for this measles epidemic. In particular, what are the transmission and recovery coefficients a and b? **HINT** for finding a: use the above rate equation for S'. Plug in S', S, and I at $t = 15$, and solve for a. (To find, or at least estimate, $S'(15)$, note that you *know* how much S changes from day 15 to day 15.1.) Express your values of a and b to six decimal places.

Later, we will use MATLAB to chart the evolution of this epidemic.

2. Pictured below are two graphs depicting evolution of diseases that progress according to the usual SIR model. For both graphs, the initial values $S(0)$, $I(0)$ and $R(0)$, and the transmission coefficient a, are the same. But the two graphs correspond to different recovery coefficients b.





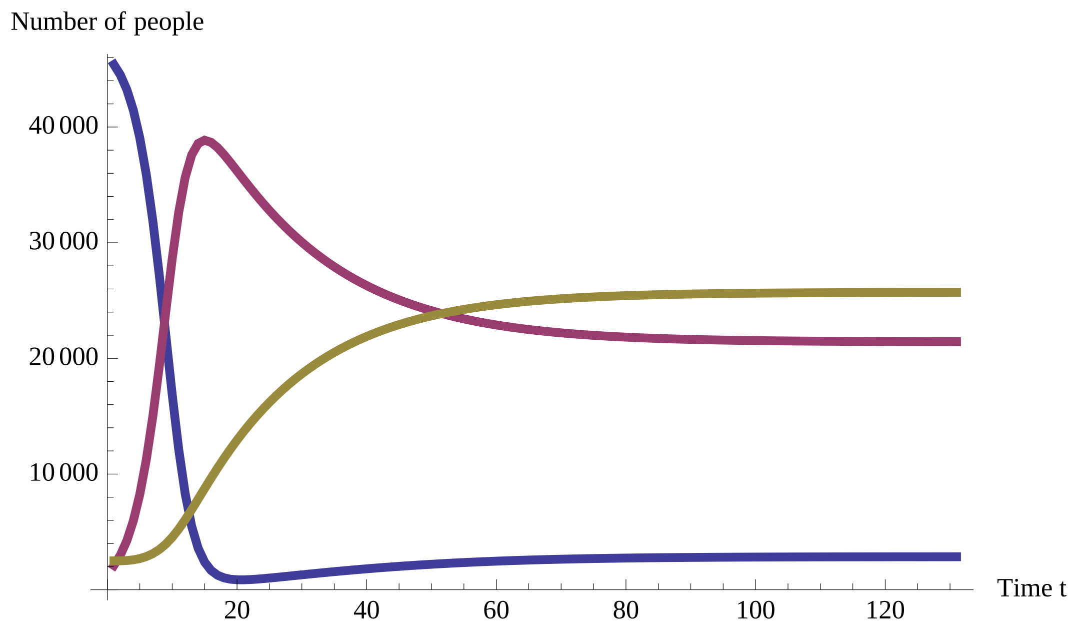
a. On each of the graphs, label which curve is S, which is I, and which is R.

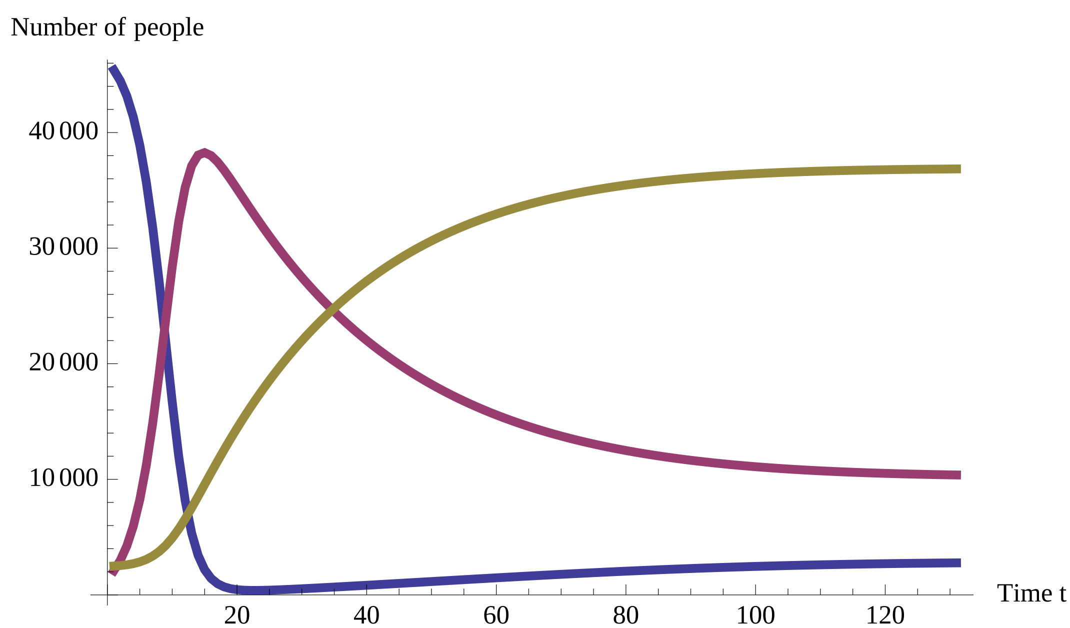
b. Which curve corresponds to the *larger* value of b? Please explain.

3. Consider an epidemic that progresses according to the usual SIR model, *except* that, now, recovered people become susceptible again (and can infect again) after c days.

a. *Modify* the usual SIR equations to reflect this new feature (wherein recovered can become susceptible again). (Your equations will involve unspecified parameters a, b, c.)

b. In the two graphs on the next page, the transmission and recovery coefficients a and b are the same, but the number of days c that it takes to become susceptible again differs from one graph to the next. For which of the two graphs – the one on the top or the one on the bottom — does it take *longer* to become susceptible again? Please explain.


Number of people
40 000
30 000
20 000
10 000
20
40
60
80
100
120
Time t


Number of people
40 000
30 000
20 000
10 000
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Time t