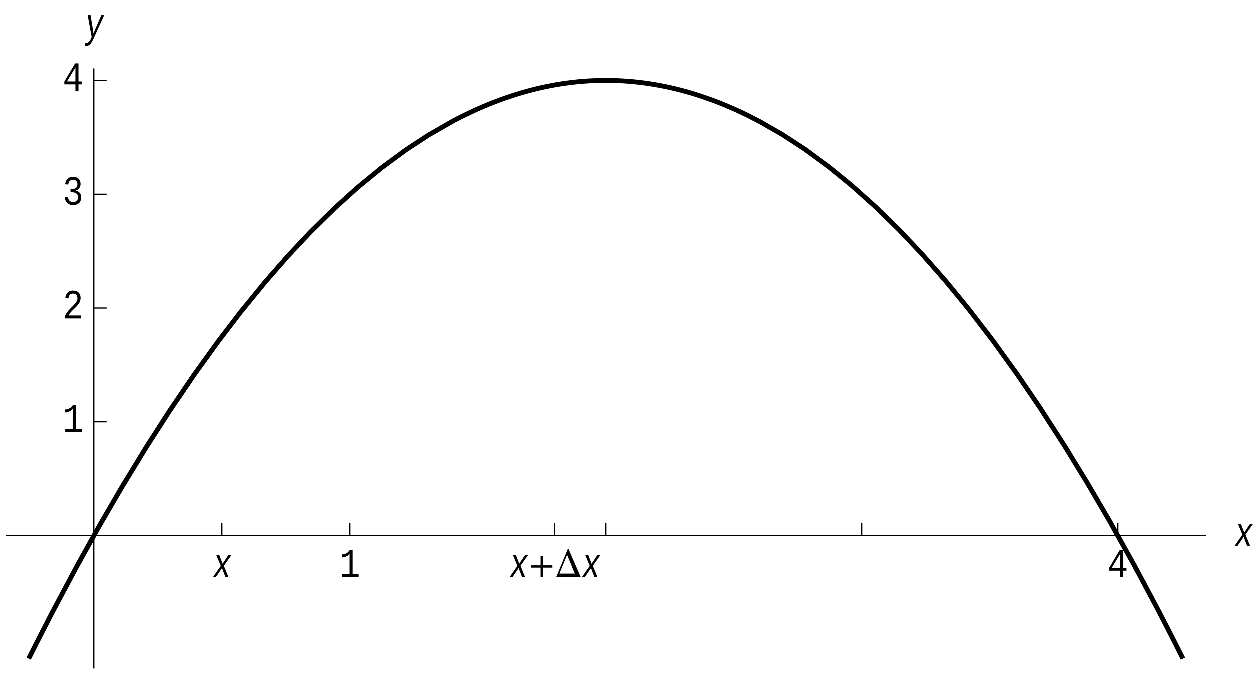
Math 408R:UT **Worksheet 5: Secant and tangent lines; the derivative 9/16/2014**

On the axes below is the graph of the function $f(x) = 4x - x^2$.



1. Where (for which values of x) is the graph of $f(x)$ increasing? Where is it decreasing?

2. Carefully draw, on the above graph,

   (a) the secant line to the graph of $f(x)$, through the points $(x, f(x))$ and $(x + \Delta x, f(x + \Delta x))$;

   (b) the tangent line to the graph of $f(x)$, at the point $(x, f(x))$.

3. Fill in the blanks: as Δx approaches __________, the above secant line becomes the ________________ line, and the slope $\Delta y/\Delta x$ of this secant line therefore becomes the slope $f'(x)$ of the ________________ line. We call this slope the __________________ of the function $f(x)$ at the point x.

   In other words,

$$f'(x) = \lim_{x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(\underline{\hspace{3cm}}) - f(x)}{\Delta x}.$$

   In still other words: as ____________ $\to 0$, the average rate of _______________ $\Delta y/\Delta x$ becomes the ______________________ rate of change $f'(x)$.

4. Let's do some computations, OK? __________________.

   Do the *algebra* (oh no, algebra?) (Yes, algebra!) required to complete the following calculation of average rate of change, for the above function $f(x)$. The answer you get at the end *should* be $4 - 2x - \Delta x$.

$$\text{average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \frac{4(x+\Delta x) - (x+\Delta x)^2 - (4x - x^2)}{\Delta x}$$

5. Let's do some *easier* computations, OK?

   ____.

   Use your answer to exercise 4 above to complete the following (your final answer should be $4 - 2x$):

$$\text{instantaneous rate of change} = \lim_{\Delta x \to 0} (\text{average rate of change})$$

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

6. Fill in the blanks: To summarize what you learned above, if $f(x) = 4x - x^2$, then $f'(x) =$ ______________.

7. Where (for which values of x) is the function $f'(x)$ you found in exercise 6 above positive? Where is it negative? (Answer using only the formula for $f'(x)$ you found above.)

8. What do exercises 1 and 7 above have to do with each other?

9. A car travels in a straight line, and its position, measured in miles s to the east of some starting point, after t minutes, where t is a number between 0 and 4, is given by

$$s(t) = 4t - t^2.$$

(a) What is the car's velocity, in miles per minute?

(b) When is the car's velocity positive, and when is it negative? What does it mean, in terms of the particulars of this situation, to say that the velocity is negative?

(c) At what point in time is the car furthest from the starting point? Please explain.