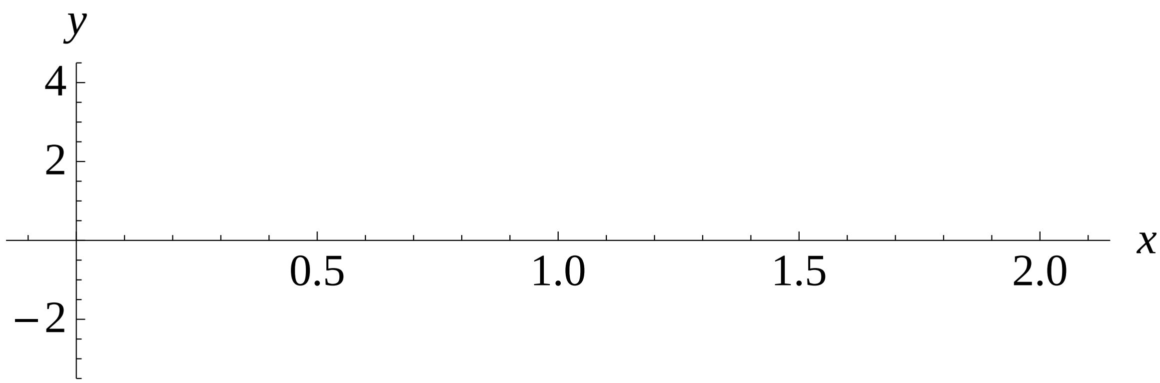
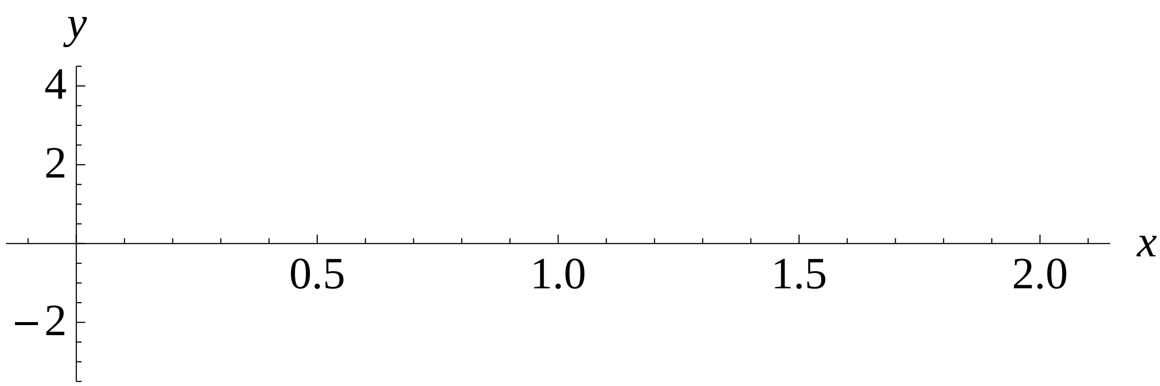
September 18, 2014 **Worksheet 6: Functions and derivatives**

Goal: To reinforce relationships among functions, their derivatives, rates of change, and slopes of tangent lines

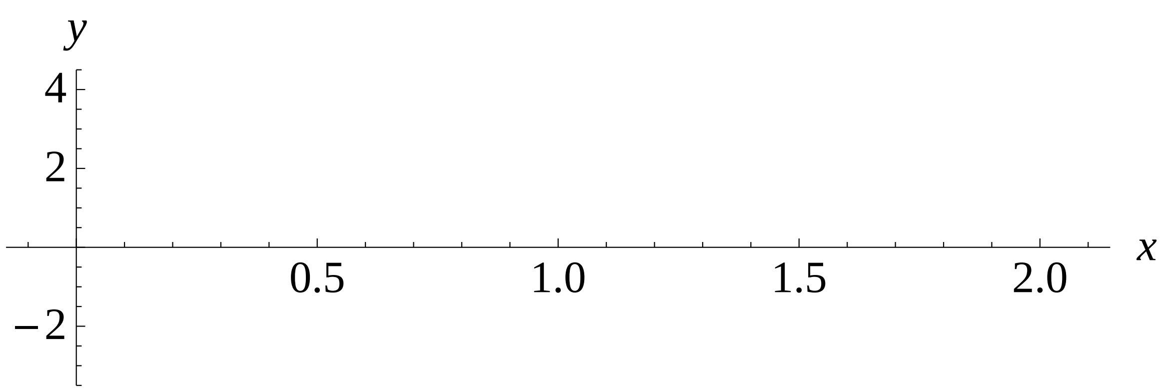
1. (a) Sketch the graph of a function f such that (i) $f'(x)$ exists everywhere on (0,2); (ii) $f(1) = 1$; and (iii) $f'(1) = 2$.



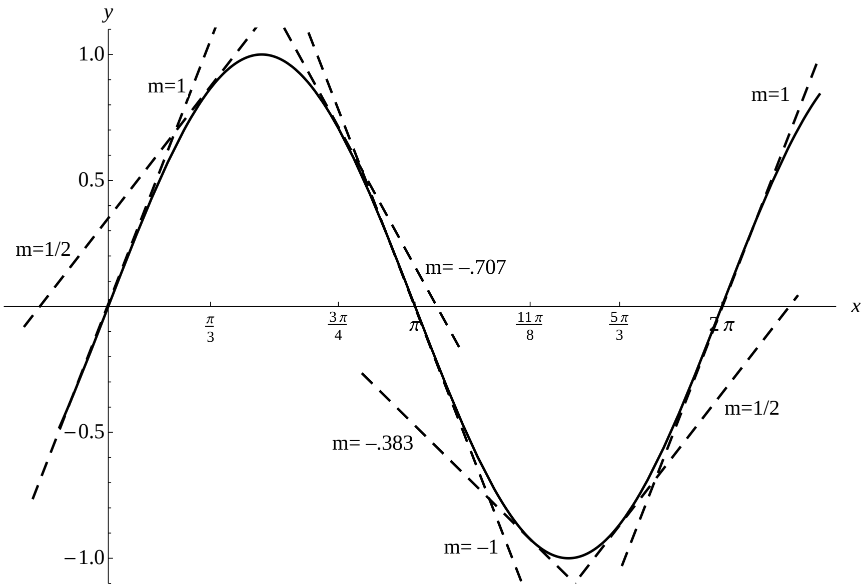
(b) Sketch the graph of a function f such that (i) $f'(x)$ exists everywhere on (0,2); (ii) $f(1) = 1$; (iii) $f'(1) = 2$; and (iv) $f(1.1) = -3$.



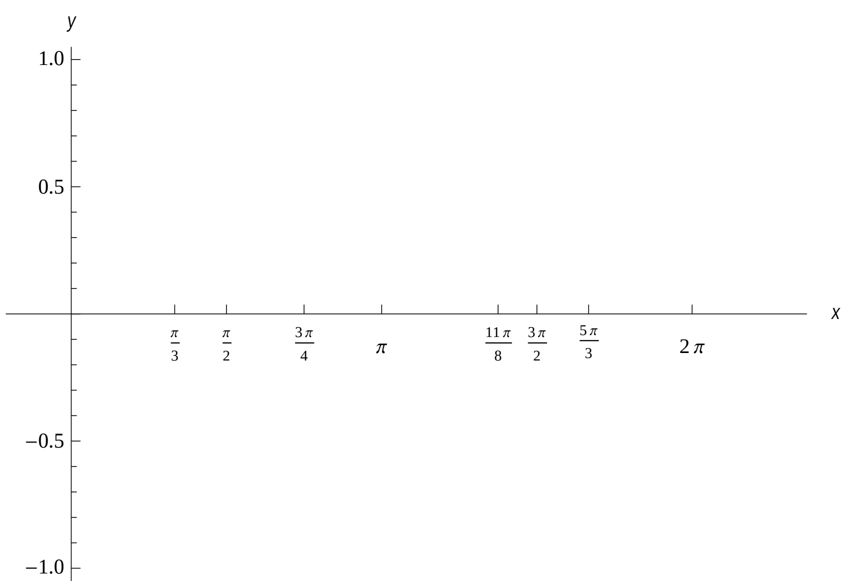
(c) Sketch the graph of a function f such that (i) $f'(x)$ exists everywhere on (0,2), *except* at $x = 1.5$; (ii) $f(1) = 1$; (iii) $f'(1) = 2$; and (iv) $f(1.1) = -3$.



2. The graph of $f(x) = \sin(x)$ is sketched below, as are the graphs of tangent lines to f at the points $x = 0, \pi/3, 3\pi/4, \pi, 11\pi/8, 5\pi/3$, and 2π. The slope of each tangent line is identified with the notation "m=..." next to that line.



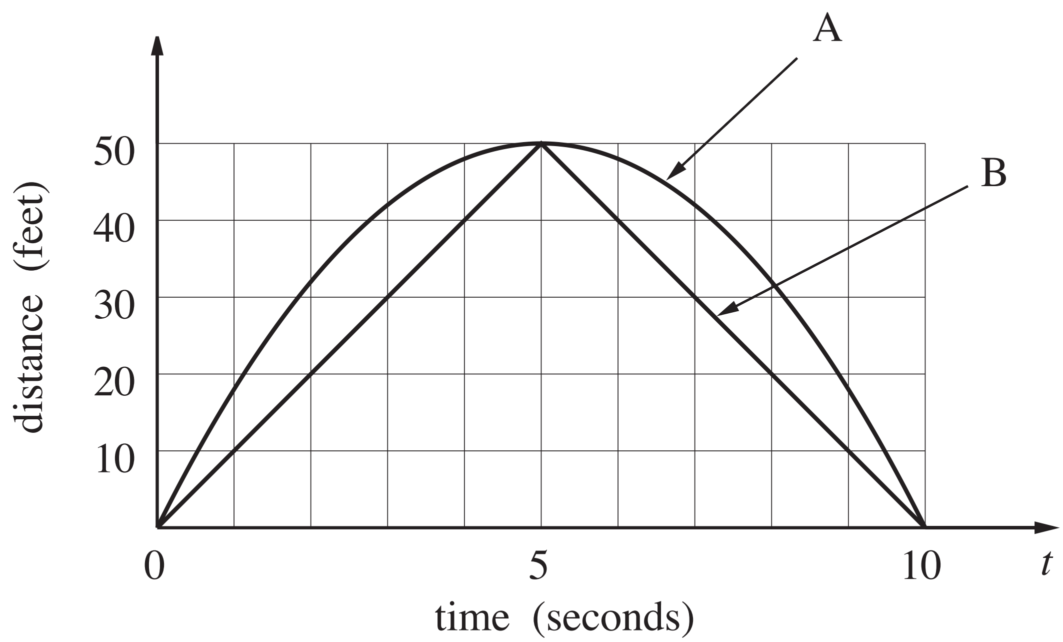
On the axes below, plot each of the above seven x values on the x axis against the slope of the corresponding tangent line on the y axis. Then connect the dots as *smoothly* as you can. (It may help to also observe that, at $x = \pi/2$ and $x = 3\pi/2$, the tangent lines to the above graph are *horizontal*.)



Based on the shape of the graph you got in problem 1 above, conjecture (guess):

If $f(x) = \sin(x)$, *then* $f'(x) =$______________________________.

3. A and B start off at the same time, run to a point 50 feet away, and return, all in 10 seconds. A graph of distance from the starting point as a function of time for each runner appears below. It tells where each runner is during this time interval.



(a) Who is in the lead during the race?

(b) At what time(s) is A farthest ahead of B? At what time(s) is B farthest ahead of A?

(c) Estimate the velocities of A and B during each of the ten seconds. Be sure to assign negative velocities to times when the distance to the starting point is shrinking. Use these estimates to sketch graphs of the velocities of A and B versus time.