September 25, 2014 **Worksheet 7: Derivative practice**

Goal: To practice using differentiation formulas and rules (sum rule; constant multiple rule; chain rule)

1. **Basic derivatives.** Find each of the following derivatives. At the end of each exercise, in the space provided, indicate which rule(s) (sum and/or constant multiple) you used. If you used a rule more than once, state how many times you used it.

   Example. Find $f'(x)$ if $f(x) = 3x - 4x^2 + 5 \cdot 17^x$.

   SOLUTION. $f'(x) = 3 - 8x + 5k_{17} \cdot 17^x$.

   Rules used: Sum rule (twice); constant multiple rule (three times).

   (a) Find $f'(x)$ if $f(x) = 5\sin(x) + 4\cos(x) - \dfrac{1}{27}\tan(x)$.

   Rules used: ____________________

   (b) Find $\dfrac{d}{dx}\left[x^5 + 5^x + \sqrt[5]{x} + \dfrac{1}{x^5}\right]$.

   Rules used: ____________________

   (c) Find $\dfrac{d}{dw}\left[\pi\cos(w) + \cos(\pi) + \pi^\pi + \pi w^\pi\right]$.

   Rules used: ____________________

2. (For this problem, you don't have to state which rules you used; just do the math.)

   (a) Find $g'(x)$ if $g(x) = 2x^3 + 7x^2 - 12x + 5$.

   (b) Find $g''(x)$ (which is called the *second derivative* of $g(x)$, and just means the derivative of $g'(x)$) if $g(x)$ is as in part (a) above.

   (c) Find $g'''(x)$ (which is called the *third derivative* of $g(x)$, and just means the derivative of $g''(x)$) if $g(x)$ is as in part (a) above.

   (d) Find $g^{(4)}(x)$ (which is called the *fourth derivative* of $g(x)$, and just means the derivative of $g'''(x)$) if $g(x)$ is as in part (a) above.

   (e) In general, if $g(x)$ is a polynomial of degree n (that is, the highest power of x appearing in $g(x)$ is the nth power), then what do you think you get if you compute $g^{(n+1)}(x)$ (the $(n+1)$st derivative of $g(x)$)?

3. **Chain rule.** Express each of the given functions of x in the form

$$y = f(u) \text{ where } u = g(x).$$

Then use the chain rule to differentiate.

Example. Find $\dfrac{dy}{dx}$ if $y = 4^{\sin(x)}$.

SOLUTION. $y = 4^u$ where $u = \sin(x)$. So

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{d}{du}[4^u] \cdot \frac{d}{dx}[\sin(x)] \\ &= k_4\, 4^u \cdot \cos(x) = k_4\, 4^{\sin(x)} \cos(x).\end{aligned}$$

(a) Find $\dfrac{dy}{dx}$ if $y = \sin(4^x)$.

(b) Find $\dfrac{dz}{dq}$ if $z = \tan(7q^2)$.

(c) Find $\dfrac{dz}{dq}$ if $z = 7\tan^2(q)$. (Recall that $\tan^2(q)$ is shorthand for $(\tan(q))^2$.)

(d) Find y' if $y = 3^{4^x}$. (Note that 3^{4^x} means $3^{(4^x)}$, not $(3^4)^x$.)

(e) Find y' if $y = 3^{x^4}$.

(f) Find y' if $y = x^{3^4}$.