M408N First Midterm Exam, September 27, 2011

1) Compute $\sec(\sin^{-1}(3/5))$. In other words, if $\sin(\theta) = 3/5$ and $-\pi/2 < \theta < \pi/2$, what is $\sec(\theta)$?

2) If $e^{3\ln(x)} = 8$, what is x? Simplify your answer as much as possible.

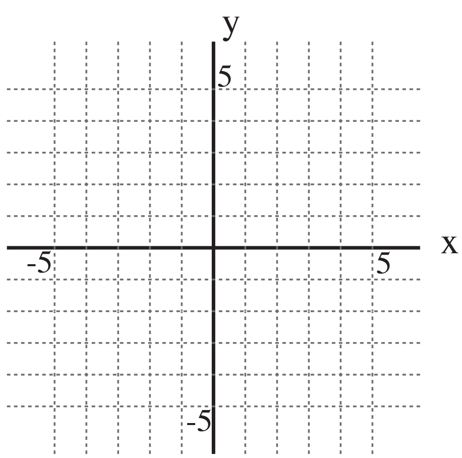
3. Compute $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x - 3}$.

4. Let $f(x) = \sqrt{2x^2 + 1}$, with a domain of $x \geq 0$. Find the formula for the inverse function $f^{-1}(x)$.

5. Consider the function $f(x) = \frac{x^2-1}{x^2-4}$. Find the vertical and horizontal asymptotes and sketch the graph $y = f(x)$.

Vertical asymptotes at:

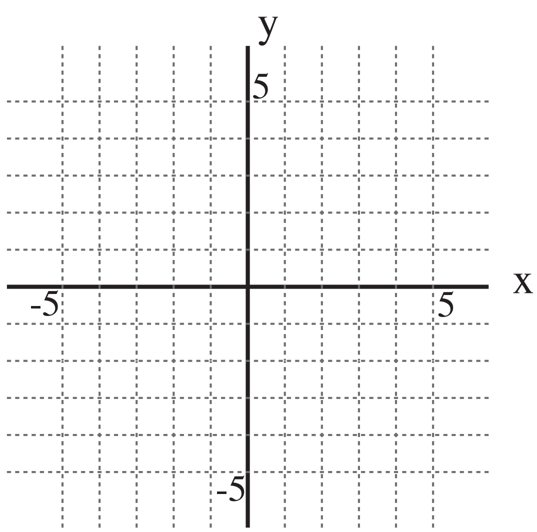
Horizontal asymptotes at:

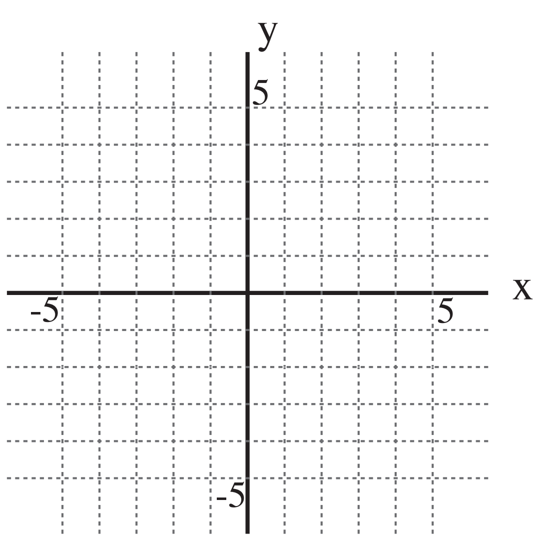


6. Consider the function $f(x) = \begin{cases} x^2 & x > 3 \\ 3x & x \leq 3 \end{cases}$. Is $f(x)$ continuous? Why or why not?

7. Suppose the position of a particle at time t is given by the function $f(t) = 2^{-t}$. (a) Graph position versus time on the first blank piece of graph paper. Be as precise as possible. (b) Sketch a graph of *velocity* versus time

on the second blank piece. This graph is *not* expected to be precise, but should be qualitatively right. You do *not* need the formula for the derivative of 2^{-t} to do this! Instead, I expect you to graph the derivative of $f(t)$ based on the shape of the graph of $f(t)$.





8. Consider the function $f(x) = 5^x$. Which of the following expressions are equal to $f'(2)$? Circle *all* correct expressions — there may not be any, there may be one, or there may be more than one. For this problem (and *only* for this problem), explanations are unnecessary and will not be considered in the grading.

a) $25 \lim_{h \to 0} \dfrac{5^h - 1}{h}$

b) $2(5)^{2-1}$

c) $\lim_{x \to 2} \dfrac{5^x - 25}{x - 5}$.

d) The slope of the line tangent to $y = f(x)$ at $(2, 25)$.

9. Let $f(x) = 1/x$. Compute $f'(-4)$ **FROM THE DEFINITION OF THE DERIVATIVE AS A LIMIT**, making clear what you are doing at every step. (If you just plug into the formula for the derivative of x^n you will not get any credit.)

10. Suppose that $f(3) = 4$ and $f'(3) = -1$. Find the equation of the tangent line to $y = f(x)$ at $(3, 4)$.

For extra credit, use this tangent line to approximate $f(3.05)$.