M408N First Midterm Exam, September 20, 2012

1) (15 pts) Suppose that at a certain time there are 500 bacteria growing in a Petri dish. The population grows exponentially, doubling every hour.

a) Find a formula for the number $x(t)$ of bacteria t hours later.

b) Find a formula for t in terms of x.

c) Now suppose that the bacteria double every 20 minutes (instead of every hour). How does this change the answers to parts (a) and (b)?

2. (15 points) Compute the following quantities exactly. The answers may involve square roots, in which case you can leave your answers looking like $\sqrt{3}/7$ or $5\sqrt{2}$ (which aren't actually the answers, of course).

a) Draw a right triangle where one of the angles has a tangent of 2. Mark the lengths of the three sides clearly. Then compute the sine of that angle.

b) Now draw a right triangle involving an angle whose sine is 1/2. Mark the lengths of all three sides, and compute the secant of that angle.

c) Compute $\sin(\cos^{-1}(1/3))$.

3. (10 pts) Consider the function $f(x) = \begin{cases} 2x+1 & x < 2 \\ 4 & x = 2 \\ 7-x & x > 2 \end{cases}$

a) Compute $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2} f(x)$, if they exist.

b) Is $f(x)$ continuous everywhere? Why or why not?

4. (30 pts) Compute the following limits:

a) $\lim_{x\to -1} \dfrac{x^2-x-2}{x-2}$.

b) $\lim_{x\to 2} \dfrac{x^2-x-2}{x-2}$.

c) $\lim_{x\to 1^+} \dfrac{x}{1-x}$.

d) $\lim_{x\to (\frac{\pi}{2})^+} \sin(x)\tan(x)$.

e) $\lim_{x\to \infty} \dfrac{2x^3-x^2+17x-5}{3x^3+139x^2-47x+\pi}$

f) $\lim_{x\to -\infty} \dfrac{|x^5+3|}{x^4+2x^2+1}$.

5. (30 points) True or False (no partial credit, and no penalty for guessing)

a) If $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exist, then $\lim_{x\to a} f(x)$ exists.

b) If $f(x)$ is a polynomial, then $\lim_{x\to a} f(x) = f(a)$.

c) The statment $\lim_{x\to\infty} f(x) = -\infty$ means that whenever x is sufficiently large and positive, $f(x)$ is large and negative.

d) If $f(x)$ and $g(x)$ are continuous at $x = a$, then so are $f(x)+g(x)$, $f(x)g(x)$, and $f(x)/g(x)$.

e) $\ln(75e^2) - 2\ln(5) - \ln(3) = 2$.

f) The inverse function of $f(x) = 3e^x + 1$ is $f^{-1}(x) = \log_e(\frac{x-1}{3})$.

g) For every x where both sides are defined, $\cot^2(x) + 1 = \sec^2(x)$.

h) If $f(x)$ is continuous on the interval $[0, 4]$, then $\lim_{x\to 3} f(x)$ must exist.

i) $\log_{10}(e) = \log_e(10)$.

j) $\log_{32}(2) = 1/5$.