M408N Final Exam, December 18, 2012

1. (60 points, 3 pages) Compute the following quantities. You do **NOT** need to simplify the derivatives, but the limits and integrals and trig functions should be actual numbers, like 3 or -7.

a) $f'(x)$, where $f(x) = e^x \sin(x)$

b) $\dfrac{d}{dx}\left(\dfrac{\ln(x)+1}{\cos(x)+2}\right)$

c) $g'(x)$, where $g(x) = \tan^{-1}(x^2)$

d) $\dfrac{dy}{dx}$, where $e^x y^2 + x^2 \ln(y-2) = 42$

e) The derivative of $(x^2+1)^{\sin(x)}$ with respect to x.

f) $\sin(\tan^{-1}(5/12))$

g) $\displaystyle\lim_{x\to 1^-} \frac{(\ln(x))^2}{x^2-1}$

h) $\displaystyle\lim_{x\to \pi/2} (\sec(x) - \tan(x))$

i) $\displaystyle\lim_{x\to 1} \frac{x^2-3x+2}{x^2-1}$

j) $\displaystyle\lim_{x\to 3^-} \frac{5-x^2}{\ln(x-2)}$

k) $\displaystyle\int_1^3 3x^2 - 4x + 5\,dx$

l) $\displaystyle\frac{d}{dx}\int_1^{x^2} \sqrt{1+e^t}\,dt$

2) (10 pts) Derivatives and limits

a) Use linearization (or equivalently, differentials) to approximate $\sqrt{10}$.

b) Use one step of Newton's method to find an approximate solution to $x^2 - 10 = 0$, starting with an initial guess of $x_0 = 3$.

3. (8 points) Optimal origami

A certain math professor (who you might recognize from his bald head, glasses and mustache) likes to fold origami flowers. He decides to supplement his salary by selling these flowers at a street festival. He determines that if he sets the price at x dollars, then he can sell $300 - x^2$ flowers. At what price can he earn the most money? How many flowers will he sell, and how much money will he make?

4. (12 pts, 2 pages) Consider the function $f(x) = xe^{-x^2/2}$.

a) Make a sign chart for $f(x)$. Then compute $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ and find any vertical or horizontal asymptotes that the curve $y = f(x)$ may have.

b) Compute $f'(x)$ and find all of the critical points. Make a sign chart for $f'(x)$. For each critical point, determine whether it is a local maximum, a local minimum, or neither.

c) Compute $f''(x)$, make a sign chart for $f''(x)$, and find all the points of inflection.

d) Sketch the graph $y = f(x)$, marking clearly the local extrema, the points of inflection, and any asymptotes.

5. (10 pts) A model rocket is shot into the air. The rocket fires for 2 seconds, during which time its (vertical) acceleration is 30 (in units of meters per second squared). After that, the vertical acceleration is -10, thanks to gravity. That is

$$a(t) = \begin{cases} 30 & \text{when } 0 < t < 2 \\ -10 & \text{when } t > 2 \end{cases}$$

a) Assuming that the rocket started off motionless ($v_0 = 0$) at time $t = 0$, compute the rocket's velocity as a function of time. [You can do this by integration, anti-differentiation, or just common sense, but be sure to make the velocity continuous.]

b) How high off the ground will the rocket be at time $t = 8$? [Again, there are several ways to do this.]