

M408N Final Exam, December 12, 2015

1) Grab bag (50 points). Compute the following quantities:

- a)  $f'(x)$ , where  $f(x) = (x^2 + 3x) \ln(x)$ .
- b)  $\frac{dg}{dt}$ , where  $g(t) = \frac{\cos(t)}{t^2 + 1}$ .
- c) The derivative of  $\sin(\ln(x^2 + 2))$  with respect to  $x$ .
- d)  $\lim_{x \rightarrow 3} \frac{\frac{1}{3} - \frac{1}{x}}{x - 3}$ .
- e)  $\lim_{x \rightarrow 1} \frac{2e^{x-1} - 2}{\ln(x)}$ .
- f)  $\lim_{\theta \rightarrow \frac{\pi}{2}} (\sec(\theta) - \tan(\theta))$ .
- g)  $\frac{d(x^{1/x})}{dx}$ .
- h)  $\int_2^3 3x^2 - 4x + 1 \, dx$ .
- i)  $\frac{d}{dx} \int_{-2}^x \frac{t^2 e^{-t} dt}{1 + t^4}$ .
- j)  $\frac{d}{dt} \int_{-3t^2}^{e^t} \cos(se^s) ds$ .

2. Continuity and differentiability. Consider the function

$$f(x) = \begin{cases} x & x < -1 \\ 0 & x = -1 \\ \frac{x^2+x}{x+1} & -1 < x < 0 \\ -2x & 0 \leq x < 1 \\ \frac{x}{x-2} - 1 & 1 \leq x < 2 \\ 42 & x = 2 \\ \frac{x}{x-2} - 1 & x > 2 \end{cases}$$

This function is obviously continuous and differentiable away from the four points (-1, 0, 1, and 2) where the formula changes. This question is about what happens at those four points.

- a) List all points where the function is discontinuous? (There may be more than one.)
- b) For each of these points, indicate what kind of discontinuity the function

has.

c) List all the points at which  $f(x)$  is continuous but not differentiable.

3. Tangent lines and linear approximations. Consider the function

$$f(x) = \frac{e^{x-2}}{x}.$$

a) Find the equation of the line tangent to  $y = f(x)$  at  $(2, f(2))$ .

b) Use this line to approximate  $f(2.04)$ .

4. Local maxima and minima. Consider the function

$$f(x) = e^x(\sin(x) - \cos(x)).$$

a) Find all the critical points of  $f(x)$  on the interval  $[-4, 8]$ .

b) Use the second derivative test to determine which of these are local maxima and which are local minima. (To get full credit, you MUST use the second derivative test.)

5. Anti-derivatives. A block is moving along a 1-dimensional track with acceleration  $a(t) = 12 - 6t$ . At time  $t = 0$ , its velocity is  $v(0) = -9$  and its position is  $x(0) = 13$ .

a) Find the velocity  $v(t)$  as a function of time.

b) Find the position  $x(t)$  as a function of time.

c) At what times is the block moving forward?

d) At what times is the velocity increasing?

6. Definite integrals and Riemann sums.

a) Approximate the integral  $\int_{-2}^4 \ln(x+3)dx$  as a Riemann sum with 6 terms, using right endpoints. [You can leave your answer in terms of logs, but you should make each term explicit. That is, you might write something like “ $(\ln(20) + \ln(22) + \ln(24) + \ln(26) + \ln(28) + \ln(30))/4$ ”, but not something like  $\sum \ln(x_k + 3)\Delta x$ .]

b) Now approximate the integral  $\int_{-2}^4 \ln(x+3)dx$  as a Riemann sum with  $N$  terms, using right endpoints. Leave your answer **in  $\Sigma$  notation**.

c) Compute  $\lim_{N \rightarrow \infty} \frac{6}{N} \sum_{k=1}^N 3 \left( -2 + \frac{6k}{N} \right)^2$  by converting it to an integral and then using the Fundamental Theorem of Calculus.