M408N First Midterm Exam, October 8, 2015

1) (15 pts) (Inverse) trig functions

a) Draw a right triangle where one of the angles is $\tan^{-1}(2)$. (There are many possible answers, all with the same shape but different overall size. Pick your favorite.) Label the lengths of all three sides and then compute $\sin(\tan^{-1}(2))$.

b) Compute $\cos(5\pi/6)$.

c) Draw a right triangle where one of the angles is $\sec^{-1}(2)$. Label the lengths of all three sides and then compute $\tan(\sec^{-1}(2))$.

2. (20 points) Compute the following limits:

a) $\lim_{x\to 4} \dfrac{x^2-5x+4}{x^2-16}$.

b) $\lim_{x\to \infty} \dfrac{x^2-5x+e^{-x}}{x^2-16+3e^{-x}}$.

c) $\lim_{x\to -\infty} \dfrac{x^2-5x+e^{-x}}{x^2-16+3e^{-x}}$.

d) $\lim_{x\to 0+} \dfrac{\tan(3x)}{4x}$.

3. (15 pts) Continuity and discontinuities.

a) Where does the function $f(x) = \dfrac{x^2-9}{x^2-4x+3}$ fail to be continuous?

b) For each point where $f(x)$ isn't continuous, identify the kind of discontinuity.

4. (15 pts) Definition of derivative.

Consider the limit

$$\lim_{h\to 0} \frac{(5+h)e^{5+h}-5e^5}{h}.$$

a) Find a function $f(x)$ and a point a such that this limit equals $f'(a)$.

b) Using what you know about taking derivatives, evaluate the limit.

5. (20 pts) Compute the derivatives of the following functions with respect to x.

a) $(\sin(x) + 3)(e^x + x^2)$.

b) $\dfrac{\sin(x) + 3}{e^x + x^2}$.

c) $\sin(e^{5x} + x^2)$.

d) $\sin^2(x) + \cos^2(x)$.

6. (15 pts) Implicit differentiation.

Find the equation of the line tangent to the curve $x^2 + y^3 = 9$ at the point $(1, 2)$.