M408N Second Midterm Exam, November 12, 2015

1) Tangents and differentials

a) Find the equation of the line tangent to $y = x^{1/3}$ at (1000,10).

b) Estimate $\sqrt[3]{994}$.

2. Critical points.

a) Find all the critical points of the function $f(x) = x^2 e^{-x^2}$.

b) Classify which are local maxima, which are local minima, and which are neither.

3. Concavity. On what interval(s) is the function $f(x) = x^2 e^{-3x}$ concave up? On what interval(s) is it concave down? Identify the points of inflection.

4. Computational grab bag. Compute the following:

a) $\dfrac{d}{dx} x^{\sin(x)}$.

b) $\displaystyle\lim_{x \to 3} \dfrac{\sin(\pi x)}{x^2 - 9}$.

c) $\displaystyle\lim_{x \to 0^+} 3x \ln(x)$

d) $\displaystyle\lim_{x \to 0^+} x^{3x}$

5. Related rates. The radar gun that a cop uses does **not** actually measure the speed of a car. Rather it measures the rate at which the **distance from the cop to a car** is changing. This problem compares the two.

Suppose that a car is moving due north along a north-south road. There is a historical marker on the road. A cop is standing 300 feet east of the marker and points a radar gun at the car. When the car is 400 feet north of the marker, the gun reads "60 MPH". How fast is the car actually going?

Be sure to explain each step of your work clearly. Draw a picture. Define your variables. Write down an equation relating your variables. Use derivatives to relate the different rates of change. Solve the problem.

6. Optimization. New Balance has discovered that the number of MX623 shoes (my favorite!) that it sells depends on the price as follows: If they set the price at $\$x$/pair, then they can expect to sell $1,000,000 - 10,000x$ pairs of shoes per month. [In the real world, manufacturers like New Balance sell mostly to retail stores, which then sell to customers. For this problem, let's pretend that New Balance is selling directly to customers.]

a) At what price will their monthly **revenue** from this line of shoes be

maximized? How high can that revenue get?

b) Suppose that it costs $40/pair to make and distribute these shoes. At what price will their monthly **profit** be maximized? What is that maximum profit?