M408N First Midterm Exam, September 21, 2016

1. Precalculus

a) Draw a right triangle where one of the angles is $\sin^{-1}(3/5)$. Call this angle θ. Label the lengths of all three sides.

b) Compute $\cos(\theta)$ and $\tan(\theta)$.

c) If $8^x = 4(2^{(x^2)})$, what are the possible values of x?

2. Limits. Compute the following limits:

a) $\lim_{x\to 5^+} \dfrac{x^2-25}{\sqrt{x^2-25}}$.

b) $\lim_{t\to -2} \dfrac{t^2+t-2}{t^2-4}$.

c) $\lim_{w\to 0^+} e^w + \ln(w)$.

3. Asymptotes and continuity. Consider the function $f(x) = \dfrac{|x^3+x|}{x^3-x}$.

a) Find all the points where $f(x)$ is discontinuous.

b) Find all the horizontal and vertical asymptotes of the graph of $f(x)$.

4. Suppose that a function $f(x)$ is defined and differentiable for all x, that $f(3) = 5$ and that $f'(3) = -2$. Find the equation of the line tangent to the graph of $f(x)$ at $x = 3$.

5. Suppose that $f(x) = \ln(x^2)$. Which of the following expressions are equal to $f'(2)$? Circle ALL that apply, and explain WHY each expression is, or isn't, $f'(2)$. Don't forget the laws of logarithms!

a) $\lim_{h\to 0} \dfrac{\ln(4+4h+h^2)-\ln(4)}{h}$

b) $\lim_{x\to 2} \dfrac{\ln(x)^2-\ln(2)^2}{x-2}$

c) $\lim_{x\to 2} \dfrac{\ln((x-2)^2)}{x-2}$

d) $\lim_{x\to 2} \dfrac{\ln(x^2/4)}{x-2}$