M408N First Midterm Exam, September 21, 2016

1) The SIR model. Suppose that an epidemic is modeled by the SIR model:

$$\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI \end{aligned}$$

where the transmission coefficient is $a = 0.00025$ and the recovery coefficient is $b = 0.1$. (Here we are measuring S, I and R in people and time in days.) The initial conditions on day 0 are:

$$S(0) = 1000; \qquad I(0) = 20; \qquad R(0) = 0.$$

a) Compute $S'(0)$, $I'(0)$ and $R'(0)$.

b) Estimate $S(1)$, $I(1)$ and $R(1)$.

c) How many more people need to get sick (beyond the initial 20) before the epidemic starts to abate? [Hint: the buzzword is "threshold"]

d) Suppose that an effective public health program reduced the transmission coefficient to 0.000125. In those circumstances, how many people would need to get sick before the epidemic started to abate.

2) Rate equations. Consider the rate equation

$$y'(t) = t + 5y - y^2,$$

with initial value $y(1) = 3$.

a) Use Euler's method, with step size $\Delta t = 0.2$, to approximate $y(1.2)$.

b) Use Euler's method, with step size $\Delta t = 0.1$, to approximate $y(1.2)$.

3) Suppose that $f(x)$ is a differentiable function, and that

$$f(0.99) = 0.36418, \qquad f(1) = 0.36788, \qquad f(1.01) = 0.37154.$$

a) Estimate the value of $f'(1)$.

b) To within the accuracy of your approximation in part (1), find the equation of the line tangent to the graph $y = f(x)$ at $(1, f(1))$.

c) Estimate the value of $f(1.062)$.

4) Here is the graph $y = f(x)$ of a mystery function f between $x = -0.5$ and $x = 4$. (Generated in MATLAB, by the way) On the SAME set of axes, sketch the graph of $f'(x)$. The graph does not have to be precise, but you should clearly indicate:

- Where f' is positive.
- Where f' is negative.
- Where f' is zero.
- Where f' is relatively big, and where it's small.