M408R Third Midterm Exam, November 21, 2016

1. (30 points, 2 pages) Compute the following integrals and anti-derivatives.

a) $\displaystyle\int_1^2 \frac{x^2+3x+2}{x} dx.$

b) $\displaystyle\int \frac{6x+3\cos(x)}{x^2+\sin(x)} dx.$

c) $\displaystyle\int_0^1 6(4x-1)^2 dx$

d) $\displaystyle\int \frac{dx}{4+x^2}$

e) $\displaystyle\int_0^6 e^{-t/3} dt$

2. (20 points) Sketching accumulation functions. The following is the graph $y = f(x)$ of a function $f(x)$. Let $A(X) = \displaystyle\int_0^X f(x)dx$ be the accumulation function.

[The sketch shows a function that is positive and decreasing for $x < -2$, is zero at $x = -2$, hits a minimum at $x = -1$, increases to zero at $x = 0$, hits a local maximum at $x = 1$, drops to zero at $x = 2$, and continues down from there. This is similar in shape to the graph $y = 3x - x^3$, except that the roots are at -2, 0 and 2 instead of $\pm\sqrt{3}$ and 0.]

a) For what values of X is $A(X)$ increasing? Decreasing?

b) For what values of X is the graph of $A(X)$ concave up? Concave down?

c) Sketch the graph of $A(X)$. It doesn't have to be precise, of course, but it should have the right value at $X = 0$, should be going upwards and downwards at the right places, and should curve upwards and downwards at the right places.

3. (30 points) We are going to use integration to compute the volume of the unit ball. This ball is obtained by rotating the unit disk $x^2 + y^2 \leq 1$ in the x-y plane around the x-axis. As with the cone problem that we worked in class, we are going to slice the sphere into N more-or-less cylindrical slices, estimate the volume of each slice, add them up, and take a limit.

a) How thick is each slice (in terms of N)?

b) If we cut the sphere at a particular value of x, we get a circle. What is the cross-sectional area of this circle, as a function of x?

c) The i-th slice corresponds to an interval $[x_{i-1}, x_i]$. What is the approximate volume of this slice? (You may use left endpoints, right endpoints, midpoints, or whatever representative point you wish.)

d) Express the approximate total volume of the sphere as a Riemann sum. Be as explicit as possible.

e) Write down a definite integral, which is the limit of this Riemann sum, that gives the exact volume of the sphere.

f) Evaluate this integral with the help of the Fundamental Theorem of Calculus.

4. (20 points) Using antiderivatives. A particle is moving with acceleration $a(t) = 12t^2 - 24t + 8$, where t is measured in seconds and a is measured in feet/second^2. At time $t = 1$ it is moving with velocity $v(1) = 0$ and has position $s(1) = 5$.

a) Find the velocity as a function of time. At what times is the particle moving forward? At what times is it moving backwards? At which instants is it (momentarily) not moving?

b) Find the position as a function of time.