Mini-Project 2: SIR using Matlab, Due October 7

In this assignment, you will be asked to run, modify, discuss, and print output from the SIREulers program you have on Matlab (which may be found on our 408R MATLAB webpage, as SIREulers.m).

1. This first exercise is meant simply to make sure your SIREulers program is working properly; to get you comfortable printing Matlab output; and to get you thinking about coding using Matlab. Please read the 408R MATLAB Manual available on our MATLAB webpage. Additionally, you will find helpful information in the comments included in the m-file.

   (a) Run your SIREulers program, by entering

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   SIREulers(24,120,.00001,1/14,45400,2100,2500)
   ```

   in the Command Window (this should return three plots).

   Once you've done this, print out the three graphs and attach them to this homework. Color printing is not required.

   For the rest of this project, let's assume that our independent time variable t is measured in days, and that our dependent variables are measured in numbers of individuals.

   Printouts available separately

   (b) What are our beginning and ending values of t? What is our stepsize (recall that $stepsize = tmax/N$)? At how many total points in time will we make observations? Use the program to answer. Your answers should be *numbers*, like "23," not variable names like "tstart."

   The stepsize is 24/120=0.2.

   (c) In this model, what are the transmission and recovery coefficients, and what are the initial values of S, I, and R? On average, how long does an individual remain infected? What is the threshold value of S? Use the program to answer.

   $a = 0.00001$ *and* $b = 1/14$*, making the threshold* $b/a \approx 7143$*.*

   (d) Write down the first five values of t at which observations will be made and recorded. Also write down the last five.

   The first five are 0.0, 0.2, 0.4, 0.6 and 0.8, and the last five are 23.2, 23.4, 23.6, 23.8, and 24.0

(e) What are the indented lines of your program doing? Describe thoroughly what computations are being done, and how many times the lines of code in this loop are being executed. Estimate how long it would take you to do all of these calculations by hand (using a calculator that can only do $+$, $-$, $\times$, and $\div$). A VERY ROUGH ballpark estimate is fine, but do describe how you came up with that estimate.

The indented lines are updating the values of S, I, and R according to the rate equations and Euler's method. For instance, we need $S(n+1) = S(n) + S'(n) * dt$, but $S'(n) = -a * S(n) * I(n)$, so $S(n+1) = S(n) - a * S(n) * I(n) * dt$. If I had to do this by hand, it might take me a minute for each iteration, or about 2 hours total.

(f) In exercise (c) above, you computed the threshold value of S using information about the parameters of the epidemic. You can also determine this threshold value, at least approximately, by reading it off the graph. Do that now. (Remember: the threshold value of S is the value of S where I peaks.) What value do you get? Are the two answers you got, using these two methods, the same? If not, why not?

They're close. Eyeballing the graphs suggests that I peaks around $t = 11$ or $t = 12$, at which time S is around 7,000 or 8,000. Which is about as accurate as you might expect from roughly reading off the values of a graph without very precise markings.

2. Run your SIREulers program again, but this time, with stepsize=0.1 instead of what you computed above (and all other quantities the same as above). Print out and attach a copy of the output.

How is the output you got in this case different from that of exercise 1 (besides the fact that the dots are more closely spaced in the second figure)? (It may help to look closely at, among other things, where I peaks in each of your two figures.)

Explain why the two graphs should look different. Which of the two figures do you think is "better," in the sense of giving a closer approximation to reality?

In fact, they're not really different, since the initial stepsize was 0.2, not 0.1. The run with stepsize 0.1 is more accurate, since smaller stepsize is always better, but 0.2 was good enough that it's hard to see the difference.

From now on, we will stick with a stepsize of 0.1.

3. Run your SIREuler program again, but this time, with $b = 1/20$ instead of $b = 1/12$ (and all other quantities the same as in exercise 2 above). Print out and attach a copy of the output.

   What are the changes in the graphs of S, I, and R, relative to the graphs in exercise 2?

   The epidemic is worse. The total number of infecteds peaks a little over 30,000, instead of a little over 25,000, and the drop-off in the later stages is much slower.

   Describe in general terms; you don't have to discuss specific numerical values, although you can if you want.

   From a modeling perspective (that is, in terms of the "real life" interpretation), what's the meaning of a recovery coefficient of $b = 1/20$? Explain, from a modeling perspective, why it makes sense that changing b from $1/14$ to $1/20$ would cause changes like the ones you saw in the graphs of S, I, and R.

   Changing b from $1/14$ *to* $1/20$ *means that it takes longer for patients to recover. This both means that there are more sick people out there spreading their germs (bigger epidemic) and that it takes longer for things to settle down.*

4. Reset b to the value $b = 1/12$ of exercise 2, but now halve the transmission coefficient a (leave all other quantities the same as in exercise 2). Print out and attach a copy of the output.

   What are the changes in the graphs of S, I, and R, relative to the graphs in exercise 2? Describe in general terms; you don't have to discuss specific numerical values, although you can if you want.

   The epidemic is growing much more slowly, and doesn't seem to have even reached its peak by day 24.

   Explain, from a modeling perspective, why it makes sense that halving a would cause changes like the ones you saw in the graphs of S, I, and R.

   Less transmission means slower growth in the number of sick people.

5. Reset all parameters to the values of exercise 2. (So stepsize=0.1, $a = .00001$, $b = 1/14$.) We are now going to tweak SIREulers so that recovered become susceptible again after 10 days (this is sometimes called the SIRS model). To do this:

   (a) FIRST, save your SIREulers program under a different name, say SIRS.m. To do this you will want to have SIREulers open in you Editor Window. In the m-file, it is best to change the function name SIREulers to just SIRS (this is on line (2) of the m-file). Next use the "File..." menu at the top of your Matlab window. Under that menu, first select "Save as. You can now save this m-file as SIRS. Next, make the appropriate changes to the program, to reflect this new phenomenon where recovered become susceptible again. (**HINTS**: (a) see the solutions to exercise 3 of Tutorial #4; (b) you should only need to change two lines of code in your program.) Save and run the SIRS.m program. Print out and attach a copy of the output, formatted as in exercise 1 above.
   See separate printout

   (b) Explain what changes you made to SIREulers.m to get your program SIRS.m. You can do this by simply printing out and attaching a copy of the SIRS.m code, or you can describe the changes in a brief paragraph.
   Change the indented lines to:
   *S(n+1) = S(n) +(.1*R(n) - a*S(n)*I(n))*dt;*
   *I(n+1) = I(n) + (a*S(n)*I(n))*dt;*
   *R(n+1) = R(n) + (b*I(n) - .1*R(n))*dt;*

(c) What are the changes in the graphs of S, I, and R, relative to the graphs in exercise 2? Describe in general terms; you don't have to discuss specific numerical values, although you can if you want. Explain, from a modeling perspective, why it makes sense that the changes you made to the code would cause changes like the ones you saw in the graphs of S, I, and R.

The epidemic never ends, but seems to be settling into a steady state of 30,000 infecteds, 5,000 susceptibles and 20,000 recovereds. Every day, about 2000 infected people recover, 2000 recovered people become susceptible, and 2000 susceptible people get infected.

6. Notice that, in the graph you generated in exercise 5 above, values of I level off at a higher level than values of R. Why is this? Or to put things another way: what *single parameter* would you change, and to what would you change it, to make I and R level off at the same height? (You might change a certain parameter to make I level off at the height of R; or you might change a certain parameter to make R level off at the height of I. Either way is fine.) Explain why this makes sense from a modeling perspective.

   We introduced a "lose immunity" coefficient of $c = 0.1$, *so the rate at which recovereds become susceptible is* cR, *while the rate at which infecteds become recovered is* bI. *In the steady state, these must be the same, so we must have* $bI = cR$. *If we also want* $I = R$, *we'd better have* $b = c$. *So let's run the program again with* $b = 1/10$ *instead of* $1/14$.

   Once you've figured out how to answer the above question, go and and make the required changes to your program, and run it. Please print out the new graph, showing I and R leveling off at the same height; attach that graph to this assignment. You're done!!